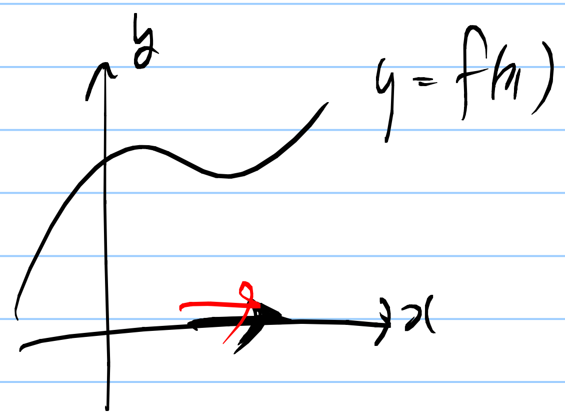
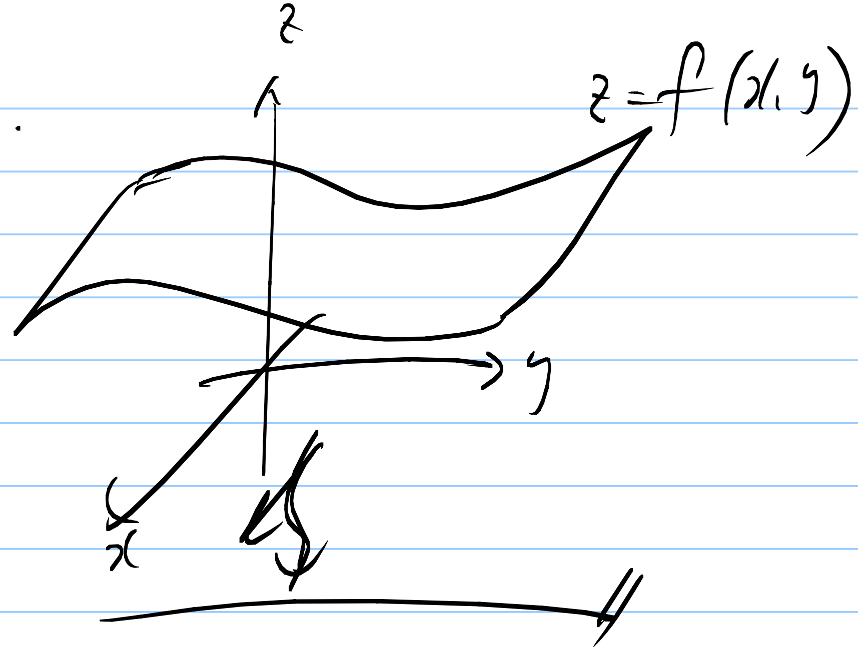
편미분 (partial derivative)

$y = f(x)$ $\qquad y' = \dfrac{df(x)}{dx}$. $\qquad \dfrac{d}{dx} f$



$$z = f(x, y) = x^2 + y^2.$$



$$g = f(x, y, z, \cdots)$$

$$\frac{\partial z}{\partial x} = 2x. \quad \frac{\partial z}{\partial y} = 2y.$$

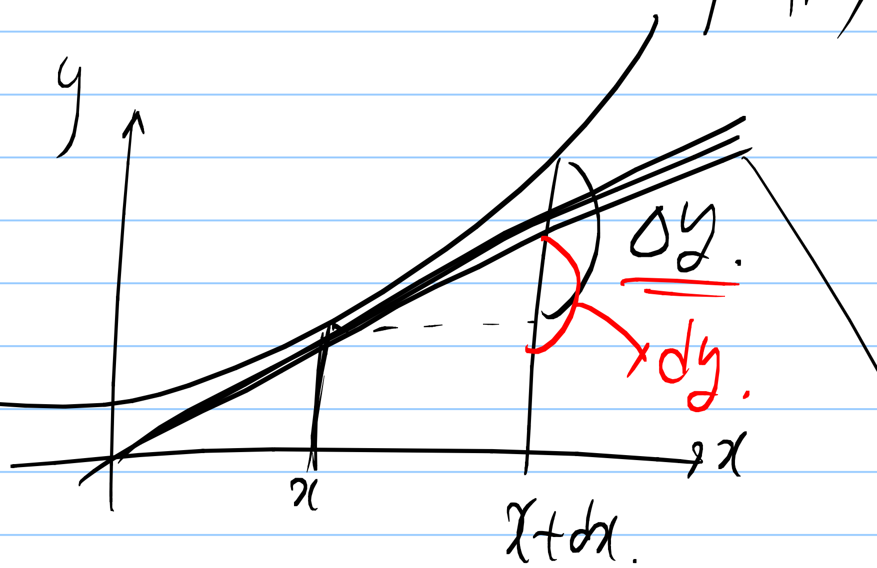
round.

∂ d.

전미분 (total derivative)

$y=f(x)$

$$dy = \left(\frac{dy}{dx}\right) \cdot dx$$



$\Delta x \to 0$

$$dy$$

$$\left(\frac{dy}{dx}\right) \cdot dx = dy$$

$$z=f(x,y) \Rightarrow dz = \left(\frac{\partial z}{\partial x}\right)dx + \left(\frac{\partial z}{\partial y}\right)dy.$$

$$(\Delta x \to 0) \quad \Delta y = f(x+\Delta x) - f(x) \approx \cancel{f(x)} + f'(x)\cdot\Delta x - \cancel{f(x)}$$

$$= \boxed{f'(x)\cdot\Delta x.}$$

$$\lim_{\Delta x \to 0.} \frac{\Delta y}{\Delta x} = f'(x) = \frac{df}{dx}.$$

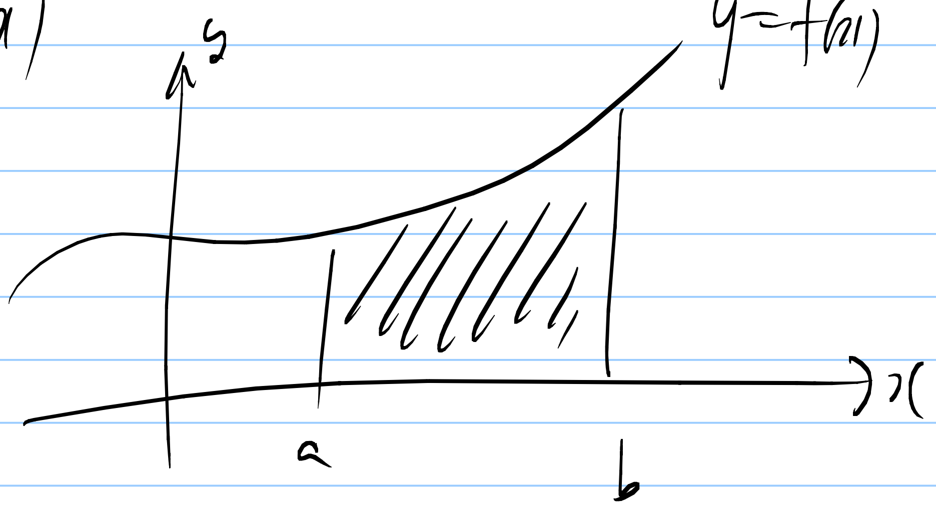
$$\Delta z = f(x+\Delta x,\ y+\Delta y) - f(x, y)$$

$$= \boxed{f(x+\Delta x,\ y+\Delta y) - f(x,\ y+\Delta x)} + f(x, y+\Delta y) - f(x, y)$$

$$= \left(\frac{\partial f}{\partial x}\right)\cdot\Delta x + \left(\frac{\partial f}{\partial y}\right)\cdot\Delta y.$$

$$dz = \left(\frac{\partial f}{\partial x}\right) dx + \left(\frac{\partial f}{\partial y}\right) dy.$$

$y = f(x)$

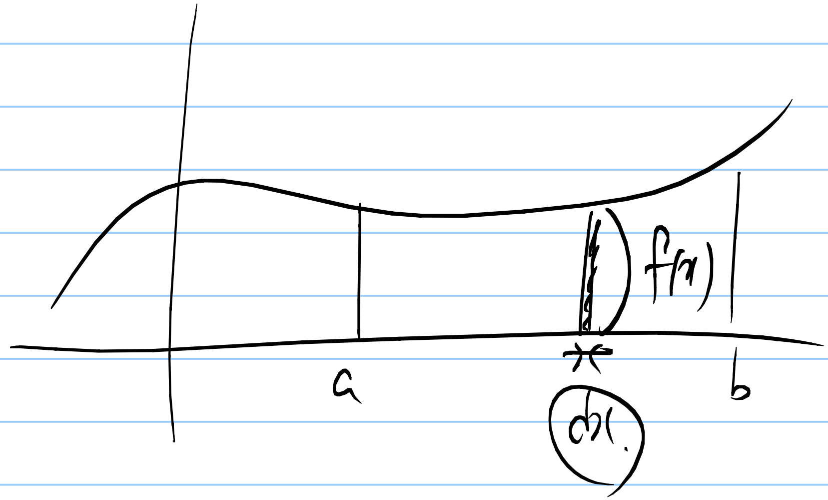


$$S = \int_a^b f(x)\,dx.$$

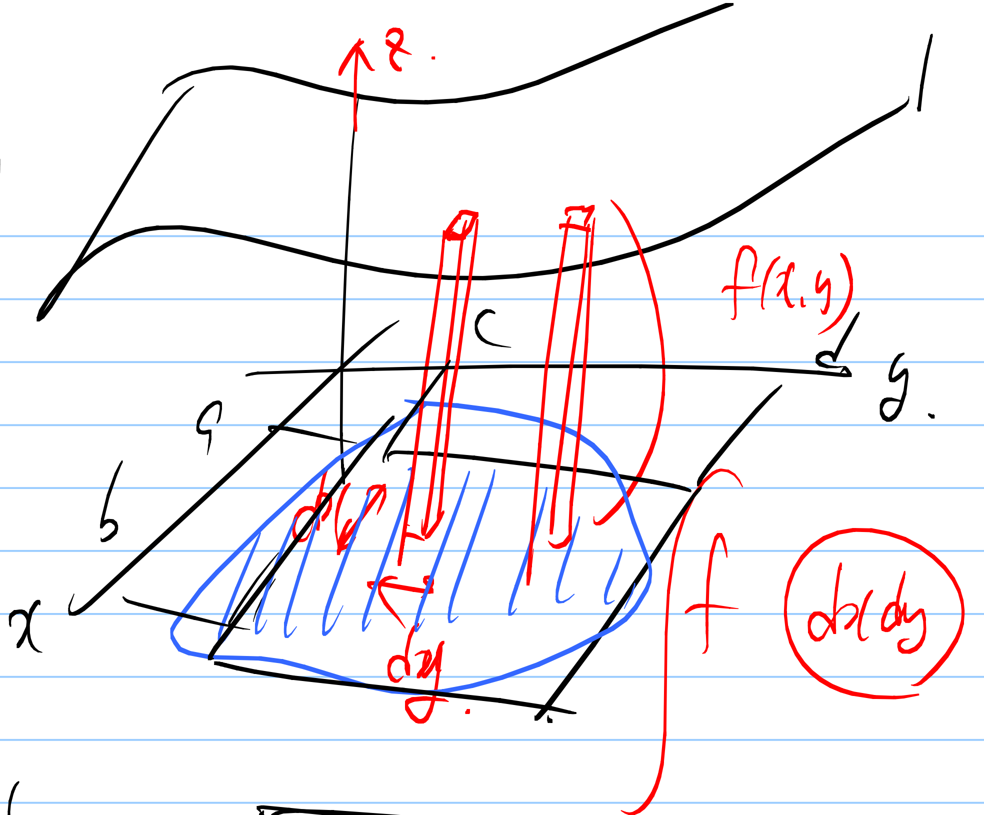
$\int f(x)\,dx.$ ⇒ 직사각형

높이

밑변.



$z = f(x, y)$

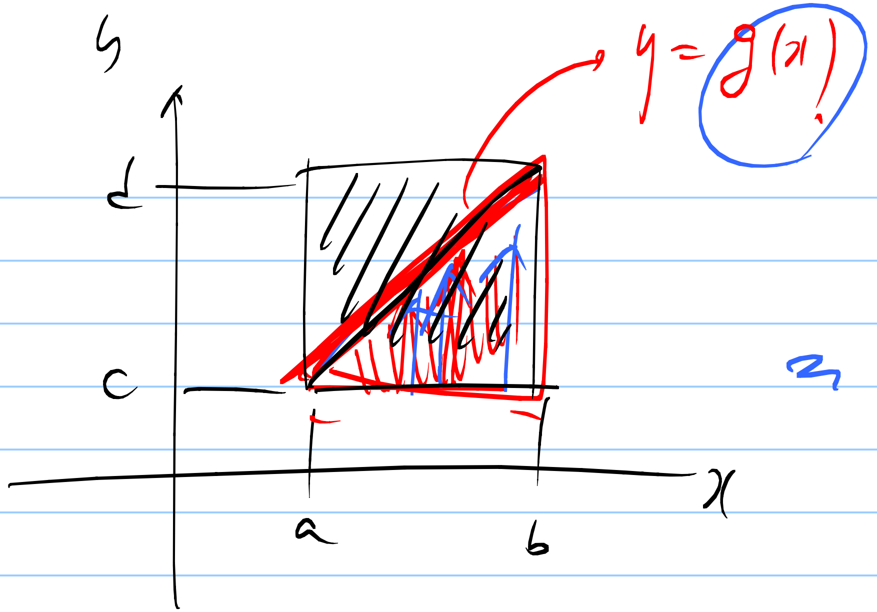


$$\int_a^b \int_c^d f(x, y)\, \boxed{dx\,dy}$$ 면적분.

$$= \int dx\,dy\; f(x, y)$$

$$= \int d^2x.$$

$z = f(x, y)$



$$\int_a^b dx \int_c^d dy$$

$$\int_a^b dx \int_c^{g(x)} dy$$

$$\iiint dx\,dy\,dz \cdot f(x,y,z) = \int d^3x \cdot f(x,y,z)$$

부피요소 $\parallel$ dV.

$$dx\,dy = dA.$$

$$\int dx\,dy\,dz\,dt = \boxed{\int d^4x}$$

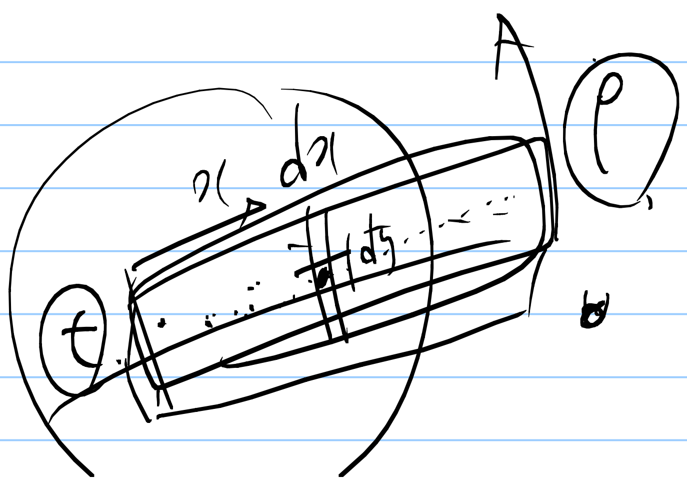
회전관성 (moment of inertia)

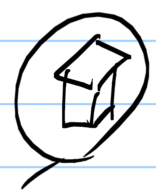
$m, v. \Rightarrow K = \frac{1}{2}mv^2.$

$\underline{v = r\omega}$ $\quad \frac{1}{2}mr^2\omega^2.$

point particle $= \frac{1}{2}(mr^2)\underline{\underline{\omega^2}}.$

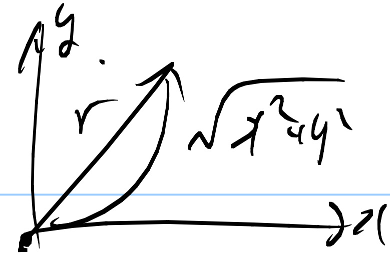
$\underline{\underline{I = mr^2}}$ $\quad = \frac{1}{2}I\omega^2.$



dm. — 

$\underline{\underline{dx\,dy\,dt\;\rho(x,y,t)}} = dm.$

$I =$ $\quad r = \sqrt{x^2+y^2}$



dm의 회전관성 $dI = dm \cdot r^2$

$$= (x^2+y^2)\,\rho\,\underline{dx\,dy\,dt}$$

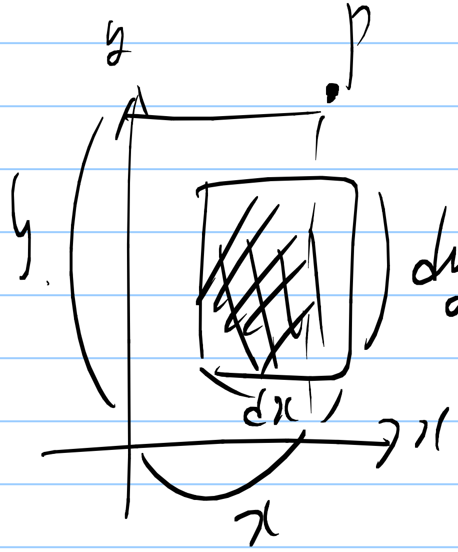
$$I = \int \boxed{(x^2+y^2)}\,\rho\,dx\,dy\,dt$$

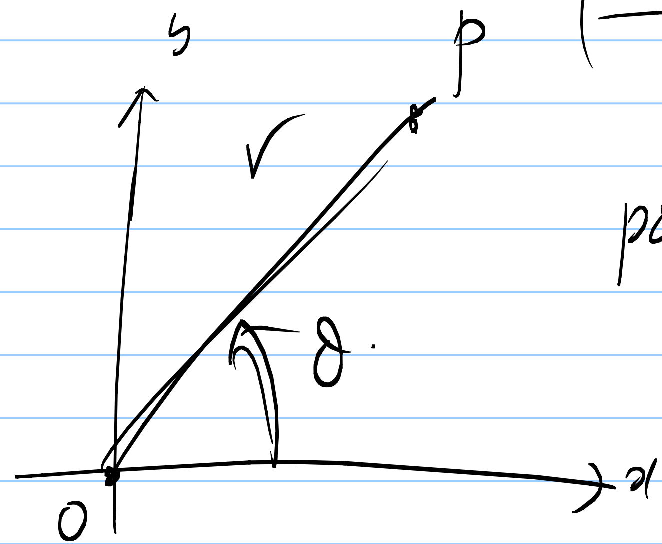
$$= \rho t \int \underline{(x^2+y^2)\,dx\,dy}$$

$y = f(x)$

$$\int dx\, f(x)$$

$x \to t(x)$





polar Coordinate.

Cartesian.

$(x, y) \to (r, \theta)$

$dx\,dy$

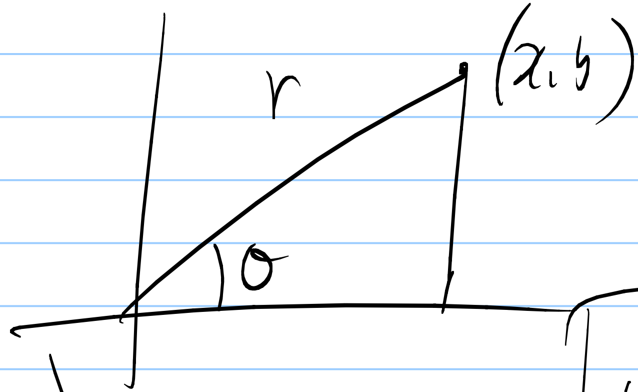
$dr\,d\theta$

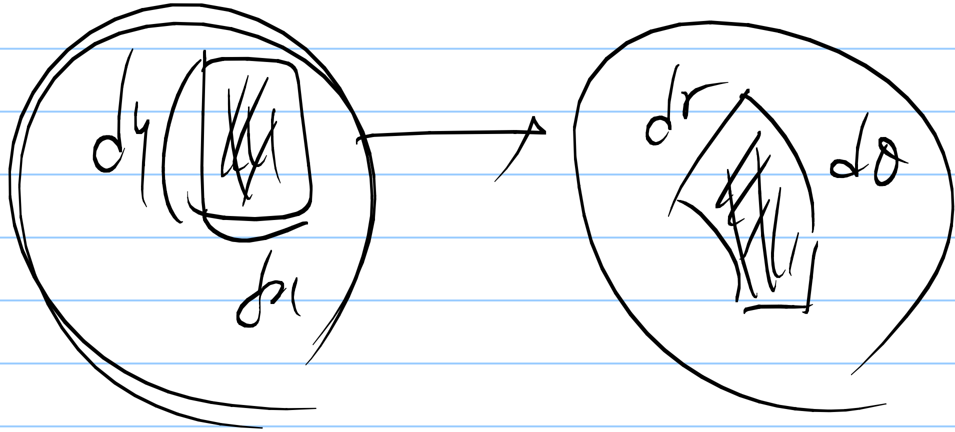
$$\int dx\,dy\, f(x,y) \stackrel{?}{=} \int dr\,d\theta\, f(x,y)$$

[길이]^2 [길이]^1

$x = r\cos\theta$

$y = r\sin\theta$.





Jacobian.

$\det\left(\quad\right)$

$$dx\,dy = \begin{vmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial x}{\partial \theta} \\ \dfrac{\partial y}{\partial r} & \dfrac{\partial y}{\partial \theta} \end{vmatrix} \; dr\,d\theta$$

$x = r\cos\theta$

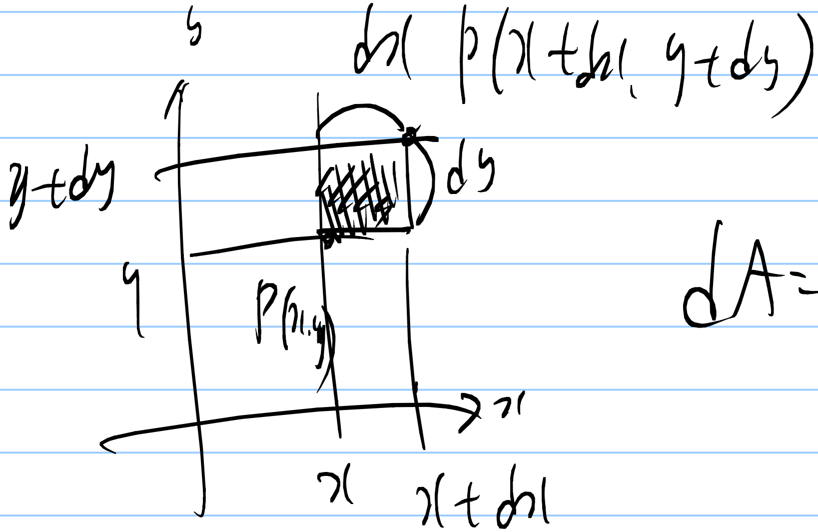
$y = r\sin\theta$

$$\frac{\partial x}{\partial r} = \cos\theta \,,\quad \frac{\partial x}{\partial \theta} = r(-\sin\theta)$$

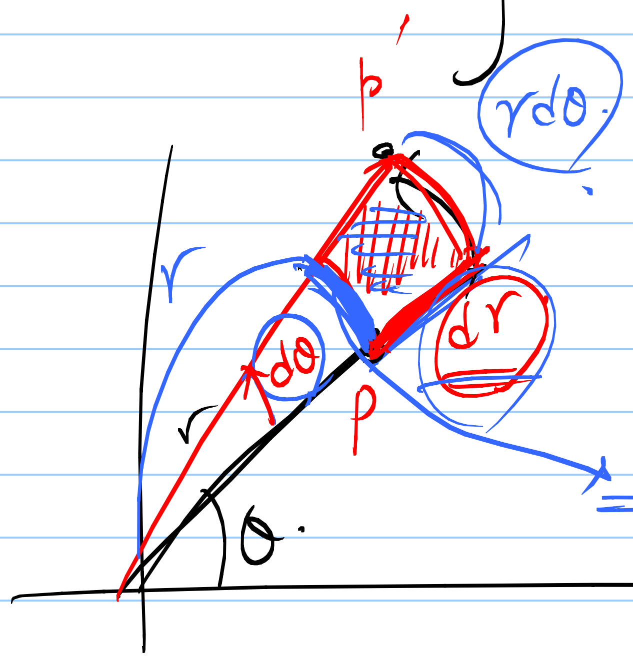
$$\frac{\partial y}{\partial r} = \sin\theta \,,\quad \frac{\partial y}{\partial \theta} = r\cos\theta .$$

$$J = r\cos^2\theta + r^2(+\sin^2\theta) = r .$$

$$\Rightarrow \quad dx\,dy = \boxed{r\,dr\,d\theta}.$$



$$dA = dx\,dy.$$

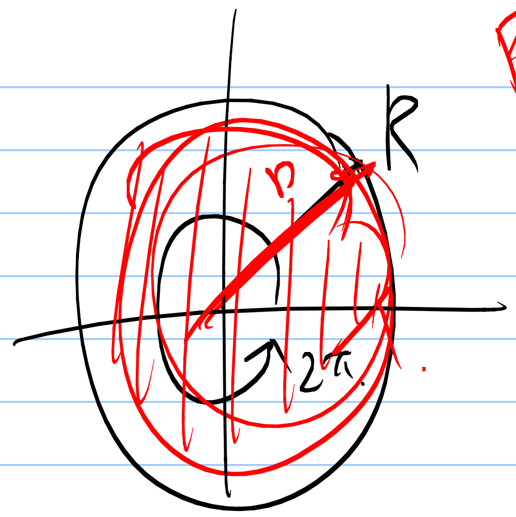


$$d\theta = \frac{\text{호의길이}}{r}$$

$$\text{호의길이} = r\,d\theta$$

$$dA = dr \cdot r\,d\theta.$$

$$= \boxed{r\,dr\,d\theta.}$$

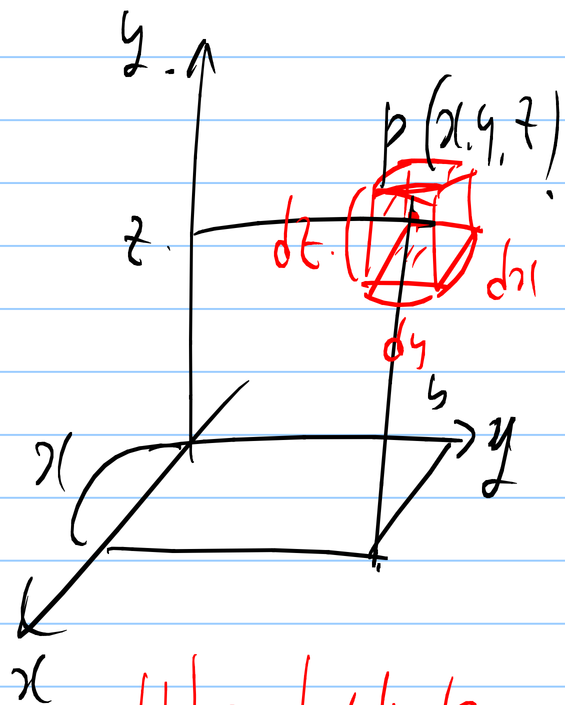


$$\int dA = \int r\,dr\,d\theta.$$

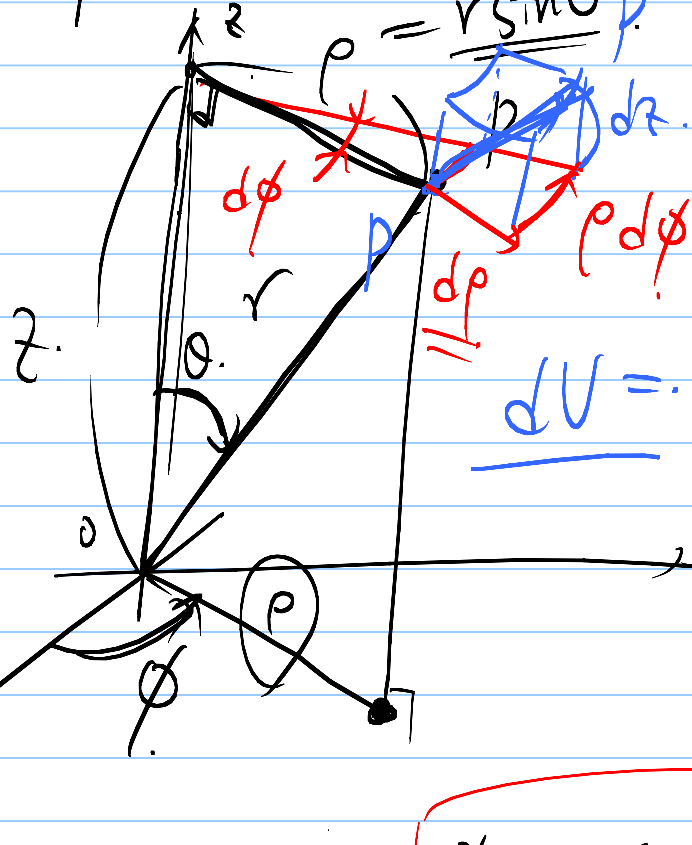
$$S = \int_0^{2\pi}\int_0^R r\,dr\,d\theta$$

$$= \int_0^{2\pi} d\theta \cdot \frac{r^2}{2}\Big|_0^R = 2\pi\,\frac{R^2}{2} = \pi R^2.$$

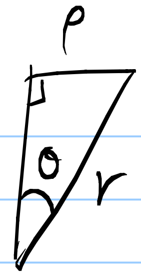
Cylindrical coordinate system.



$dV = dx\,dy\,dz.$



$\rho = \underline{r\sin\theta}$ ρ'



$\underline{dV} = d\rho \cdot \underline{\rho\, d\phi} \cdot dz.$

$\parallel$

$\underline{\rho\, d\rho\, d\phi\, dz.}$

$\underline{(x, y, z)} \rightarrow \underline{(\rho, \phi, z)}$

$$\boxed{\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \\ z &= z \end{aligned}}$$

$dV = J \cdot d\rho\, d\phi\, dz.$

$$J = \left| \frac{\partial x}{\partial \rho} \quad \frac{\partial x}{\partial \phi} \quad \frac{\partial x}{\partial z} \right| d\rho\, d\phi\, dz.$$

dU : $P \to P'$

$\rho : d\rho$. $\phi : \rho\, d\phi$. $z : dz$.

$$ds^2 = (d\rho)^2 + (\rho\, d\phi)^2 + (dz)^2.$$

$$= d\rho^2 + \rho^2 d\phi^2 + dz^2.$$

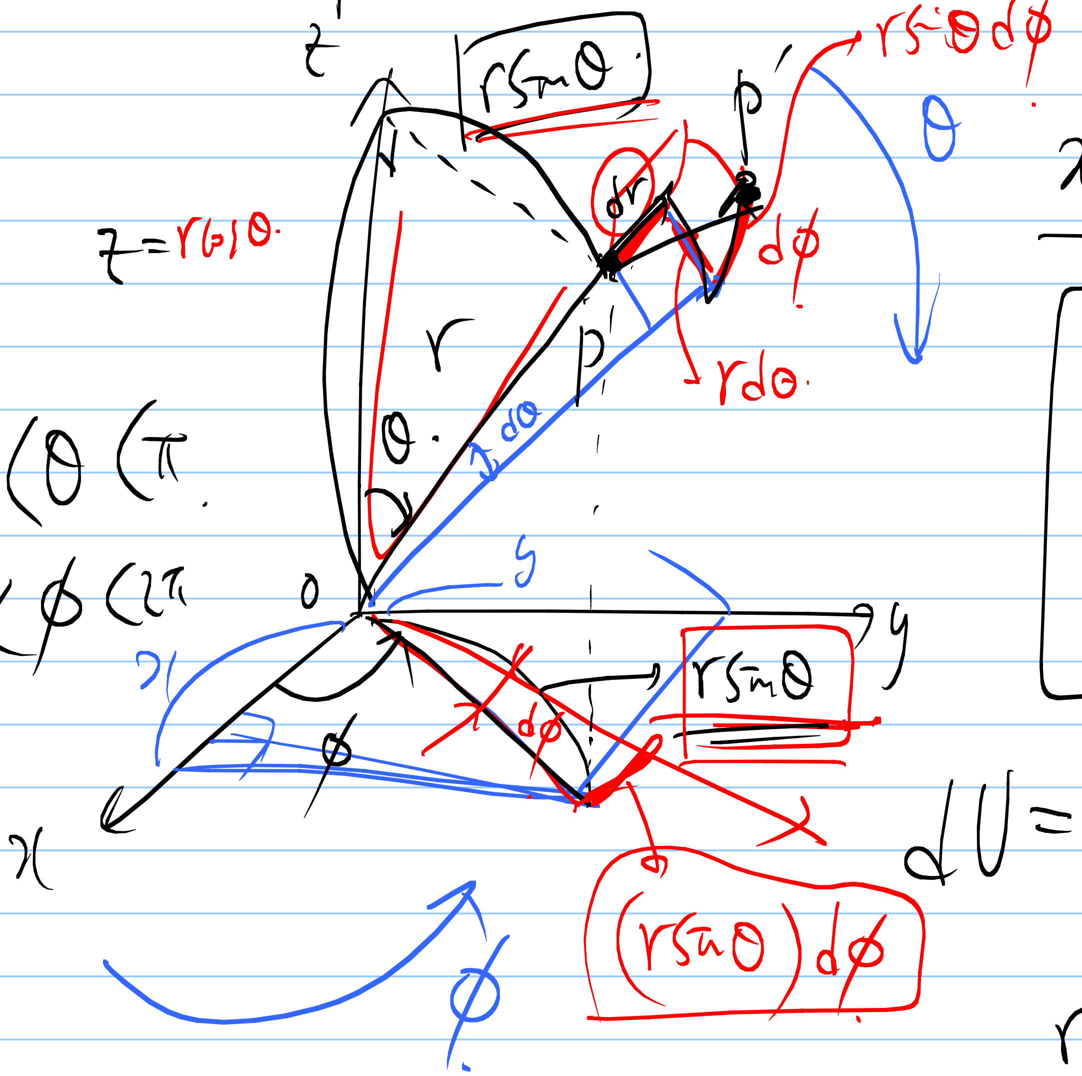
$P \xrightarrow{ds} P'$

$\boxed{ds^2 = dx^2 + dy^2 + dz^2}$.

$\Downarrow$

metric tensor

Spherical Coordinate sys. (구면좌표계)



$0 < \theta < \pi$

$0 < \phi < 2\pi$

$x, y, z \to r, \theta, \phi$

$$\begin{cases} x = \rho\cos\phi = r\sin\theta\cos\phi \\ y = \rho\sin\phi = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

$dV = dx\,dy\,dz = J\ dr\,d\theta\,d\phi$

$r : dr$, $\theta : \boxed{r\,d\theta}$ $\boxed{\phi : r\sin\theta\,d\phi}$

$= dV = dr \cdot r\,d\theta \cdot r\sin\theta\,d\phi$

$[길이]^3$

$$dV = dx\,dy\,dz = r^2 dr \sin\theta\, d\theta\, d\phi \;\; = r^2 dr \sin\theta\, d\theta\, d\phi$$

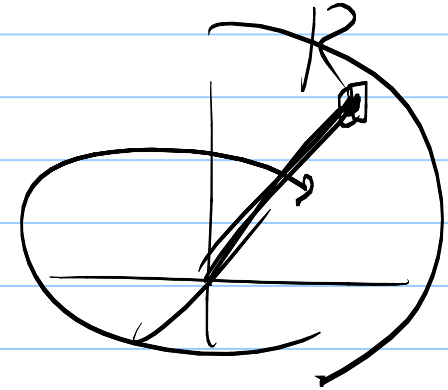
3

$$P \to P' \quad ds^2 = (dr)^2 + (r\,d\theta)^2 + (r\sin\theta\, d\phi)^2$$

$$= dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

구의 부피

$$= \int dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} r^2 dr \sin\theta\, d\theta\, d\phi$$



구. R

$$\phi\ 2\pi \int_0^{\pi} \sin\theta\, d\theta \quad \frac{1}{3} r^3 \Big|_0^R \qquad \frac{4}{3}\pi R^3$$

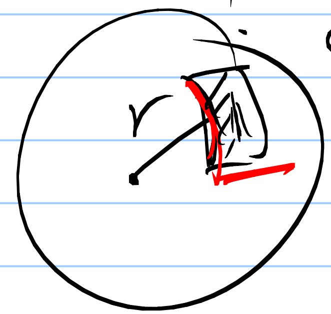
$$= 2\pi \cdot \frac{R^3}{3} \cdot \underbrace{\left[-\cos\theta\right]_0^{\pi}}_{2 = -(-1)+1} = \frac{4}{3}\pi R^3$$

$$\cancel{dr}$$

$dr = 0$

$d\theta$

$d\phi$



$$dS = r\,d\theta \cdot r\sin\theta\, d\phi$$

$$= r^2 \sin\theta\, d\theta\, d\phi$$

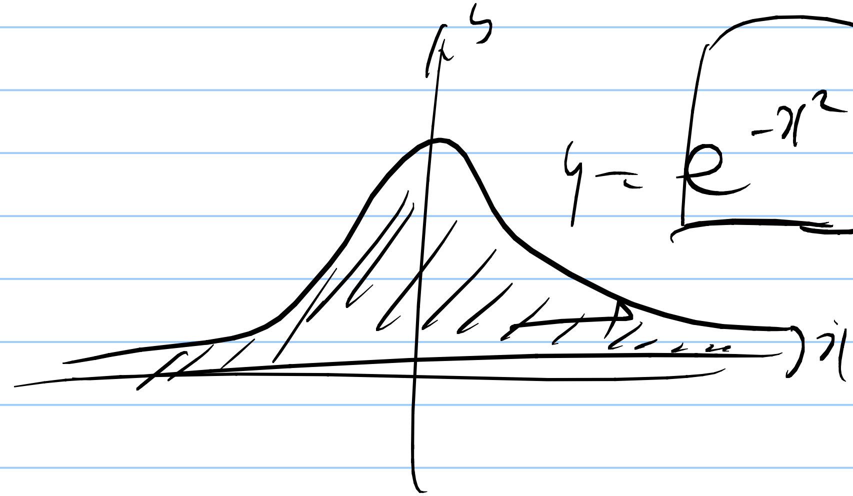
$$\int dS = r^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi = r^2 (2\pi) \cdot 2 = 4\pi r^2$$

$\boxed{\sin\theta\, d\theta\, d\phi} = d\Omega$: 입체각 (solid angle)

반지름 1인 구의 넓이 요소.

$$\boxed{\int d\Omega = 4\pi}$$

$$\boxed{\int_{-\infty}^{\infty} e^{-x^2}\, dx}$$



$y = \boxed{e^{-x^2}}$

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx \cdot \boxed{\int_{-\infty}^{\infty} e^{-y^2}\,dy}$$

$x = r\cos\theta$

$y = r\sin\theta$.

$r^2 = x^2 + y^2$.

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\,dy\; e^{-x^2-y^2}.$$

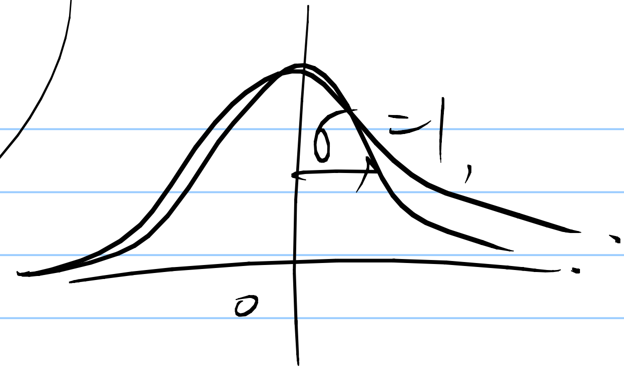
$$= \int_{0}^{2\pi}\int_{0}^{\infty} r\,dr\,d\theta \cdot e^{-r^2}$$

$r^2 = t$

$2\boxed{r\,dr} = dt$

$$= (2\pi)\cdot\left[-\frac{1}{2}e^{-r^2}\right]_0^{\infty} = 2\pi\left[-0+\frac{1}{2}\right] = \pi.$$

$$\therefore \int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}.$$



$$\int_{-\infty}^{\infty} \frac{e^{-x^2}}{\sqrt{\pi}}\,dx = 1$$

$$\int_{0}^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}.$$

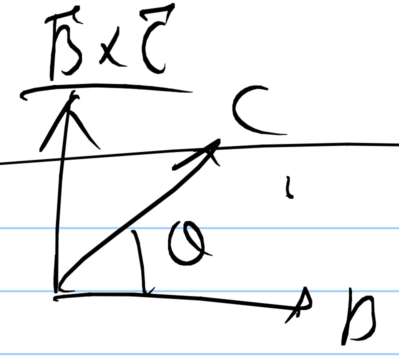
Vector Analysis.

$$\int_V \underline{\vec{\nabla}} \cdot \vec{V} \, dU = \int_{\underline{\partial V}} \vec{V} \cdot d\vec{S}$$

$$\int_S \vec{\nabla} \times \vec{V} \cdot d\vec{S} = \int_\ell \vec{V} \cdot d\vec{r}.$$

$$\vec{A} \cdot \vec{B} = \sum_{x=1}^{3} A_x B_x \qquad \underline{(\vec{A} \times \vec{B}) \cdot \vec{C}}.$$

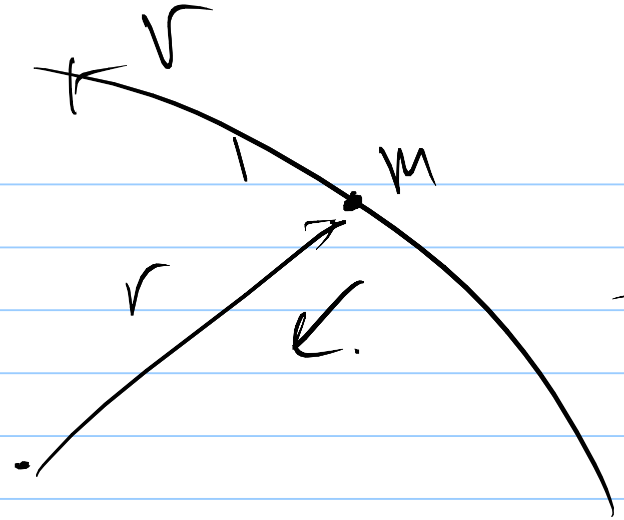
$$\vec{A} \times (\vec{B} \times \vec{C}) =$$



$\vec{B}$ 와 $\vec{C}$ 가 이루는 평면 위에 있다.

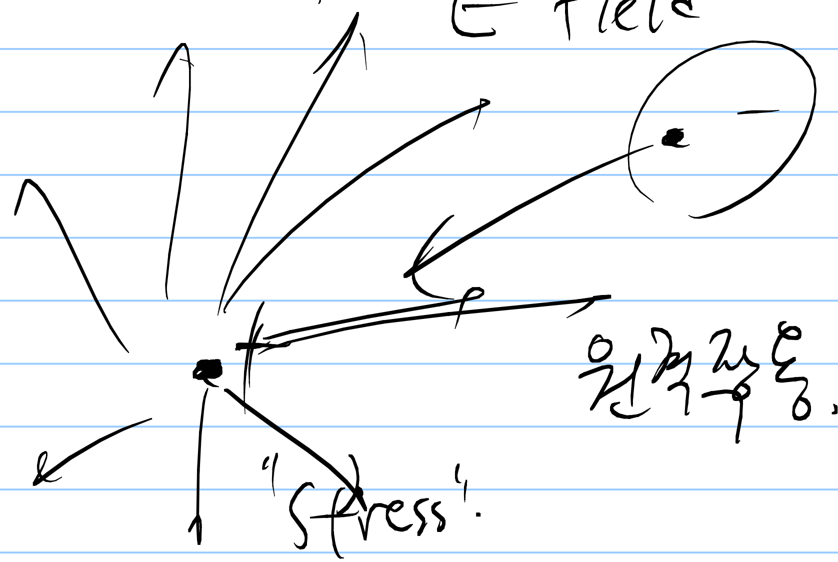
$$= (\vec{A} \cdot \vec{C})\vec{B} + \boxed{-\vec{A} \cdot \vec{B}}\,\vec{C}$$

$$\frac{d\vec{r}}{dt} = \vec{v} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right)$$



$$\frac{mv^2}{r} = m\left(\frac{v^2}{r}\right)$$

장 (field)



$$\vec{v}(x,y,z)$$

$\boxed{\phi(x, y, z)}$

$\boxed{\text{gradient}}$ "del"

$\vec{\nabla}$: 미분연산자

$\vec{\nabla}\phi$

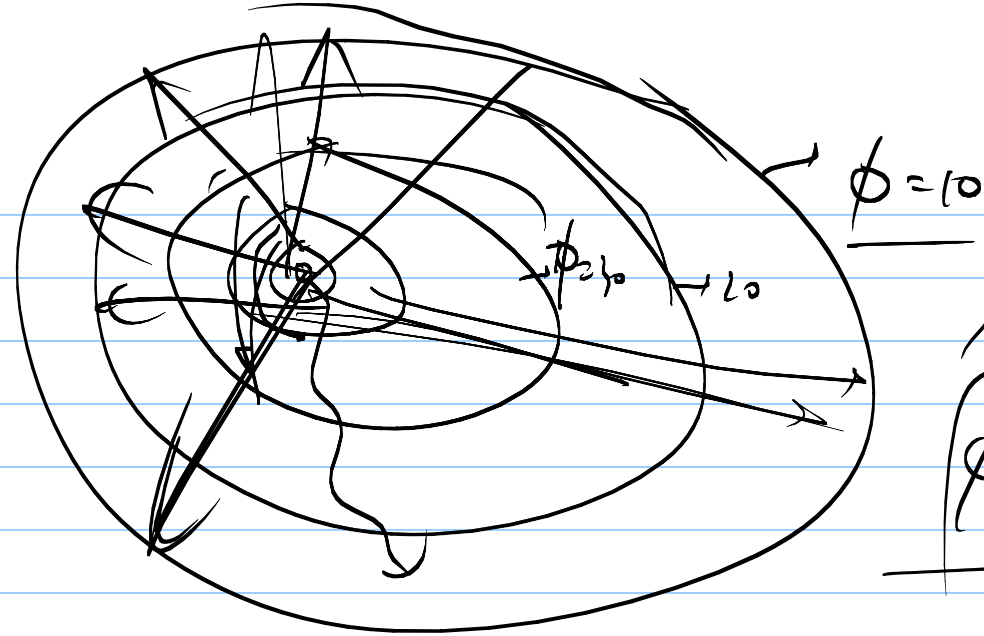
$$= \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

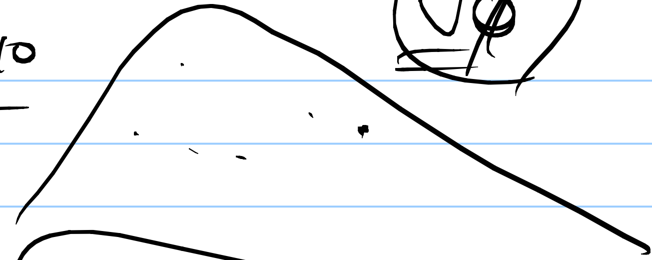
$$\boxed{\vec{\nabla}\phi} = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$: vector

방향도함수
(directional derivative)

$\vec{\nabla}\phi$

$\vec{\nabla}\phi$





$\vec{\nabla}\phi$

$\boxed{\phi(x,y,z)}$

$$\vec{\nabla}\phi = \sum_i^3 \frac{\partial}{\partial x_i}\hat{x}_i$$

$$\frac{\partial\phi}{\partial r} + \frac{\partial\phi}{r\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\phi}$$

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2\sin\theta\, d\phi^2$$

$\vec{\nabla}\phi$

$\vec{\nabla}$

$$\boxed{\vec{\nabla}\cdot\vec{U}} = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)\cdot\left(U_x\hat{x} + U_y\hat{y} + U_z\hat{z}\right)$$

$$= \frac{\partial}{\partial x}U_x + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z}.$$

$$= \sum_i^3 \frac{\partial}{\partial x_i}U_i = \mathrm{div}\,\vec{V}.$$

$$\vec{\nabla}\times\vec{U} = \mathrm{curl}\,\vec{U} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ U_x & U_y & U_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial U_z}{\partial y} - \frac{\partial U_y}{\partial z}\right) + \cdots$$

$$\vec{\nabla}\cdot(\vec{\nabla}\phi) = (\vec{\nabla}\cdot\vec{\nabla})\phi = \vec{\nabla}^2\phi = \frac{\partial^2}{\partial x^2}\phi + \frac{\partial^2}{\partial y^2}\phi + \frac{\partial^2}{\partial z^2}\phi.$$

$\square$: Laplacian.

$$\nabla^2\phi = 0 \qquad \nabla^2\phi = \frac{1}{a^2}\frac{\partial^2\phi}{\partial t^2}.$$

$$\nabla^2\phi = \frac{\partial}{\partial t}\phi,$$

$$\boxed{-\frac{\hbar^2}{2m}\vec{\nabla}^2\psi + U(\vec{r})\psi = E\psi.}$$

$$\frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{(mv)^2}{2m}.$$

운동에너지.

퍼텐셜.

$$\nabla^2 = \vec{\nabla}\cdot\vec{\nabla}$$

$$\boxed{p \Rightarrow \frac{\hbar\vec{\nabla}}{i}}.$$

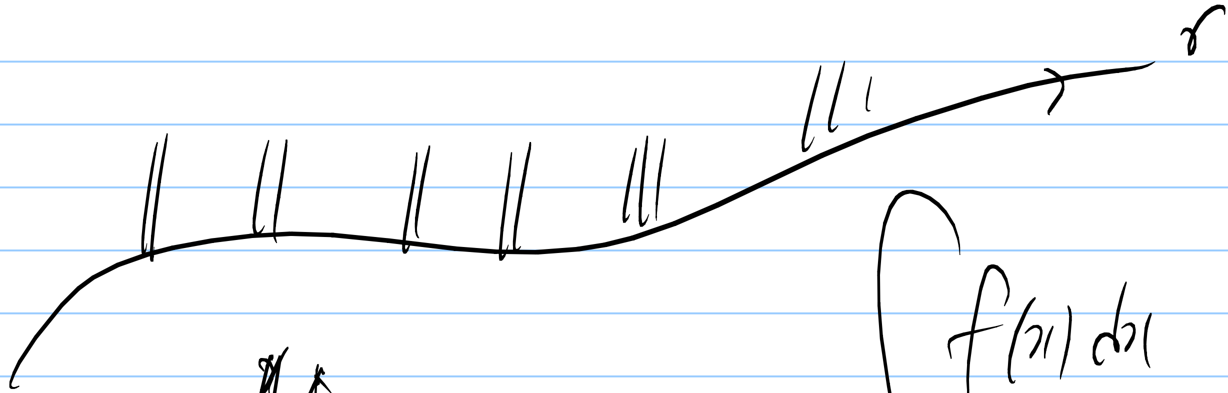
$$\vec{\nabla}\times(\vec{\nabla}\phi) = 0.$$

$$\boxed{\vec{A}\times\vec{A} = 0}$$

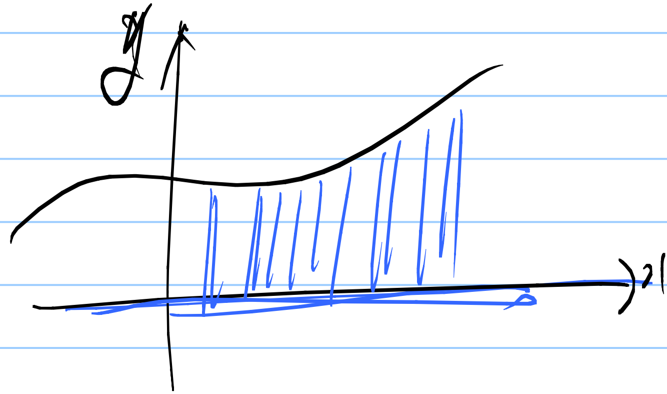
$$\vec{A}\cdot(\vec{A}\times\vec{B}) = 0.$$

$$\boxed{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{v}) = 0}$$

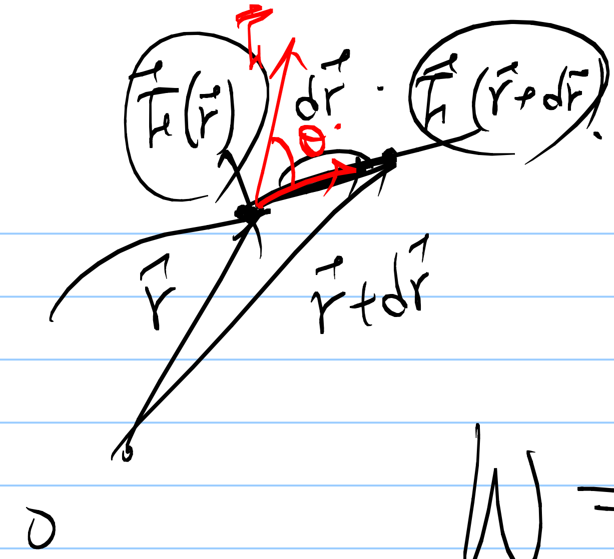
선적분 (line integral)



$$\int f(x)\,dx$$



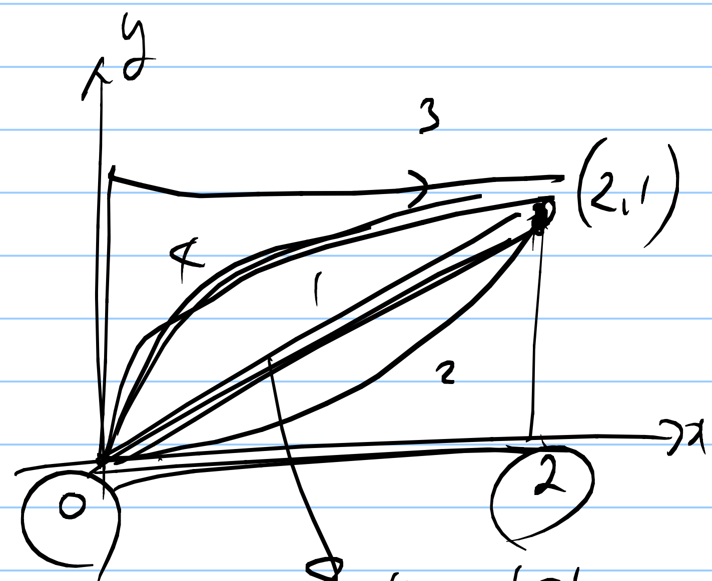
일(work) → $W = F \cdot S$ $\qquad \vec{F} = \vec{F}(\vec{r})$



$$\vec{F}(\vec{r})\cdot d\vec{r} = dW.$$

$$W = \int \vec{F}\cdot d\vec{r}.$$

$$\vec{F} = (xy, -y^2)$$



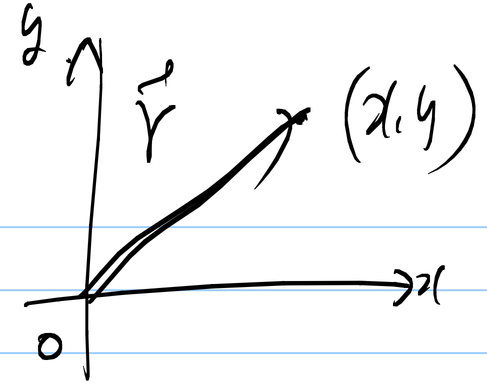
$$\vec{r} = (x, y)$$

$$y = \frac{1}{2}x.$$

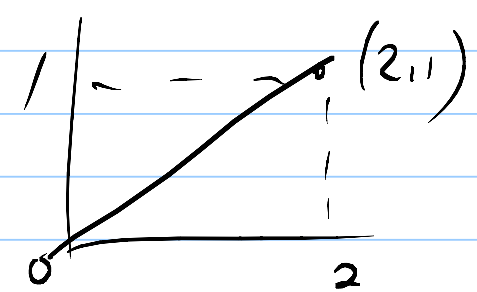
$$\int \vec{F}\cdot d\vec{r}.$$

$$d\vec{r} = (dx, dy)$$

$$W = \int_l \boxed{\vec{F} \cdot d\vec{r}} = \int_l \left[xy\,dx - y^2 dy \right].$$



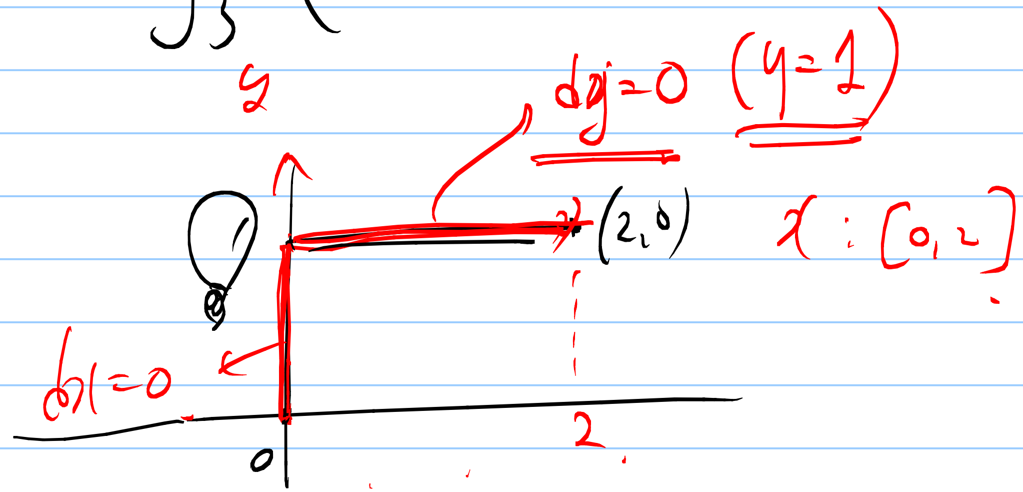
$$y = \frac{1}{2}x \; ; \; W = \int_0^2 x \cdot \frac{1}{2}x \cdot dx - \int_0^1 y^2 dy.$$



$$= \int_0^2 \frac{x^2}{2} dx - \left[\frac{1}{3} y^3 \right]_0^1$$

$$= \frac{1}{6} x^3 \Big|_0^2 - \frac{1}{3} = 1.$$

$$W = \int_3 \vec{F} \cdot d\vec{r} = \int_3 \left(xy\,dx - y^2 dy \right)$$

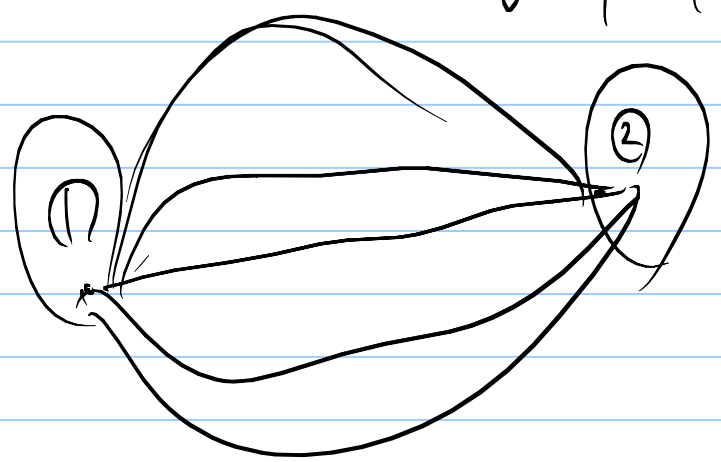


$$= 0 - \underline{\int_0^1 y^2 dy} + \int_0^2 xy\,dx \Big|_{y=1}$$

$$= -\frac{1}{3} + 2 = \frac{5}{3}$$

$\vec{F}$

$W_1 \neq W_3$ path-dep.



$\vec{F}$: 비보존력. (non-conservative)

$\vec{F}$: 보존력. $\longrightarrow$ $\boxed{\vec{\nabla} \times \vec{F} = 0}$

$$W = \int_{1}^{2} \vec{F} \cdot d\vec{r} = \int_{V_1}^{V_2} dV = V_2 - V_1.$$

$\boxed{\vec{F} \sim \vec{\nabla}\phi}$

$\vec{\nabla} \times \vec{\nabla}\phi = 0.$

$$\int dx = x. \qquad \int df = f$$

$$\int f' dx = f = \int \frac{df}{dx} \cdot dx = \int df$$

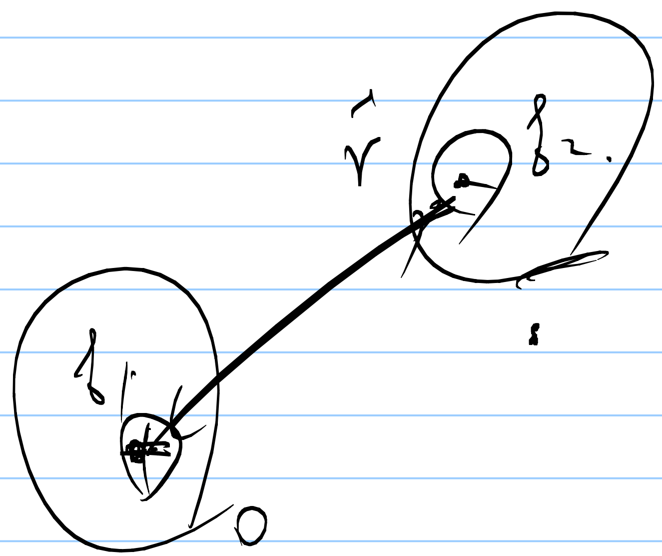
보존력 : $\nabla \times \vec{F} = 0 \rightarrow \vec{F} = -\nabla \phi$.

potential.

$$\boxed{V = \int -\vec{F} \cdot d\vec{r}.}$$

$\oplus \leftrightarrow \ominus$

$$\vec{F}_c = \frac{1}{4\pi\epsilon_0} q_1 q_2 \cdot \frac{r\hat{r}}{r \cdot r^2} = \frac{1}{4\pi\epsilon_0} q_1 q_2 \cdot \frac{\vec{r}}{r^3}$$



$$= q_2 \vec{E}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} q_1 \frac{\vec{r}}{r^3}$$

Electric field.

$$\vec{\nabla} \times \frac{\vec{r}}{r^3} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{r^3} & \frac{y}{r^3} & \frac{z}{r^3} \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F}_c = 0.$$

$$\frac{\vec{r}}{r^3}$$

$$r = \sqrt{x^2+y^2+z^2}.$$

보존력 $\vec{F}$: $\vec{\nabla} \times \vec{F} = 0$.

$\Rightarrow \boxed{\vec{F} = -\vec{\nabla}\phi}$

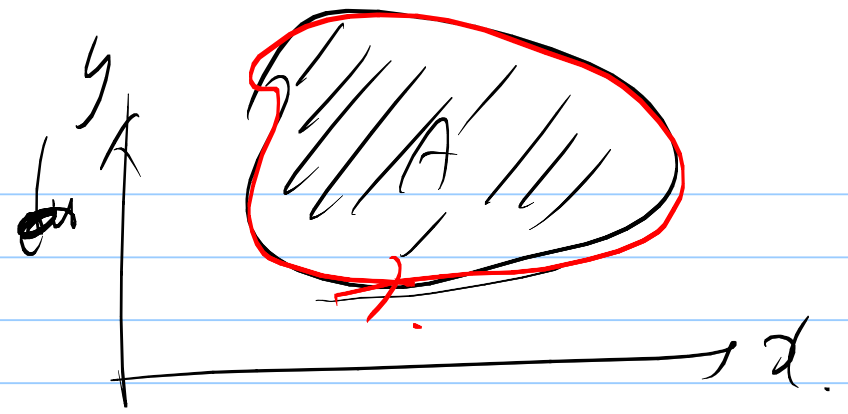
ϕ : potential

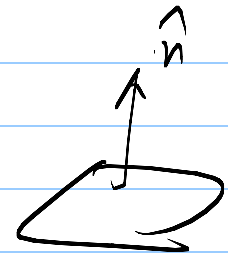
$$\int f' \, dx = f.$$

$$= \int \frac{df}{\cancel{dx}} \cancel{dx} = \int df = f.$$

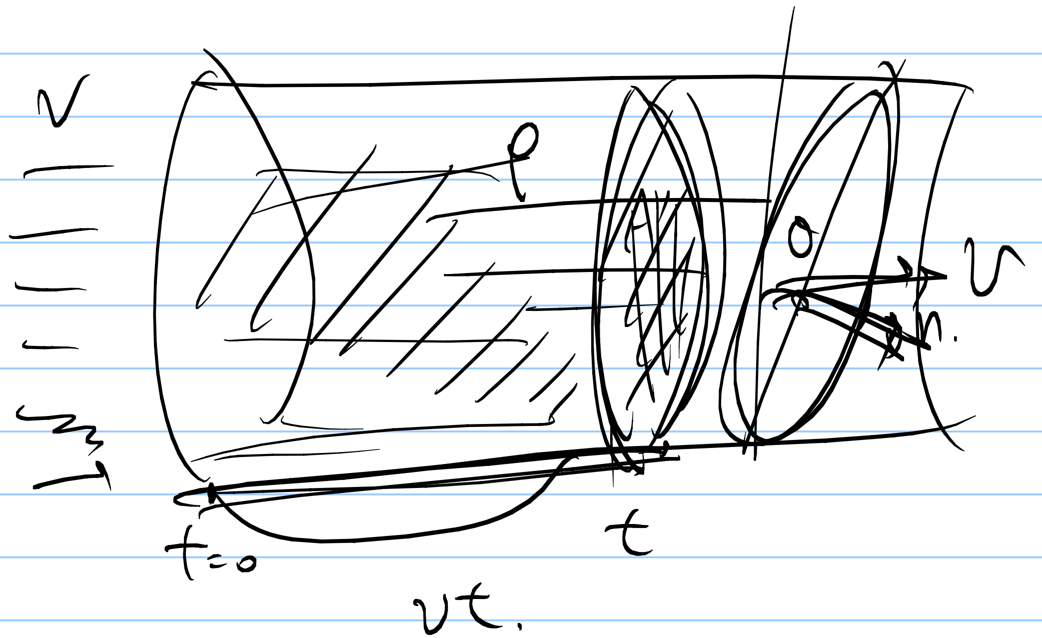
$$\int_{A} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \boxed{dx\,dy} = \int_{\partial A} (P\,dx + Q\,dy)$$

$d\alpha$





$$\boxed{\underline{\vec{\nabla}\cdot\vec{V}} \ : \text{divergence}} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}.$$



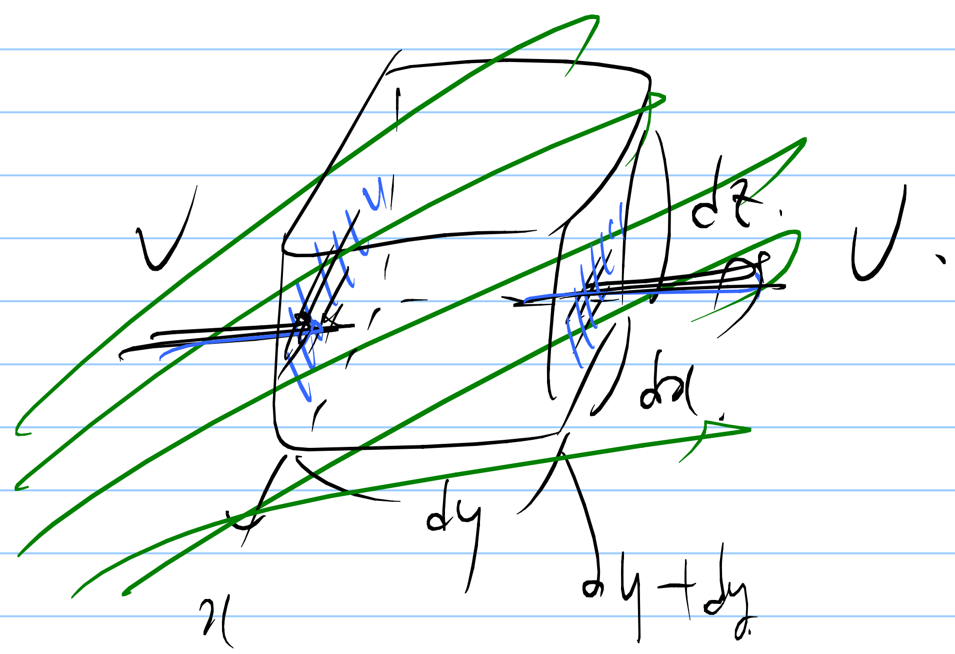
$V = vt \cdot A.$

$$\boxed{vt \cdot A \cdot \rho.}$$

$\vec{v}\cdot A\hat{n} =$

$$\underline{vt \cdot A' \cdot \rho} \Rightarrow \boxed{\vec{v} \cdot \underline{A\hat{n}}\, t\rho} \frac{1}{t}$$

$$\boxed{\rho\, \vec{v} \cdot \hat{n}} = \vec{U}\,\hat{n}$$



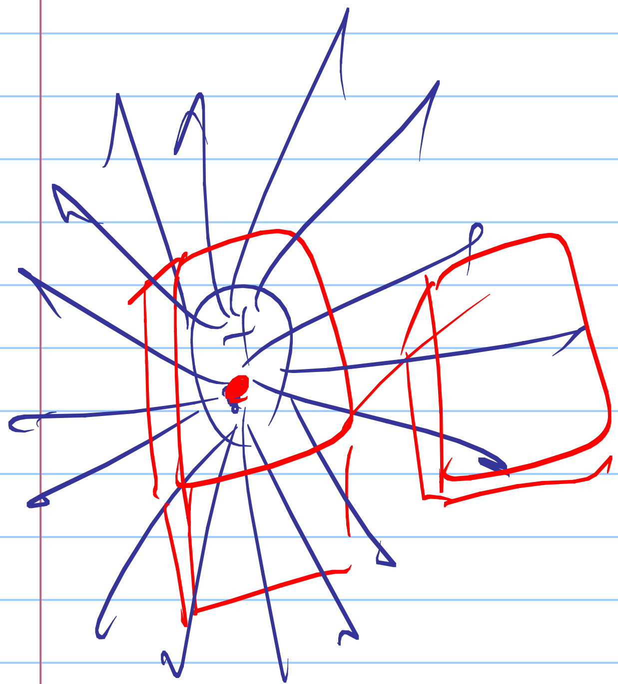
$$\left[v_y(y+dy) - v_y(y) \right] dxdz.$$

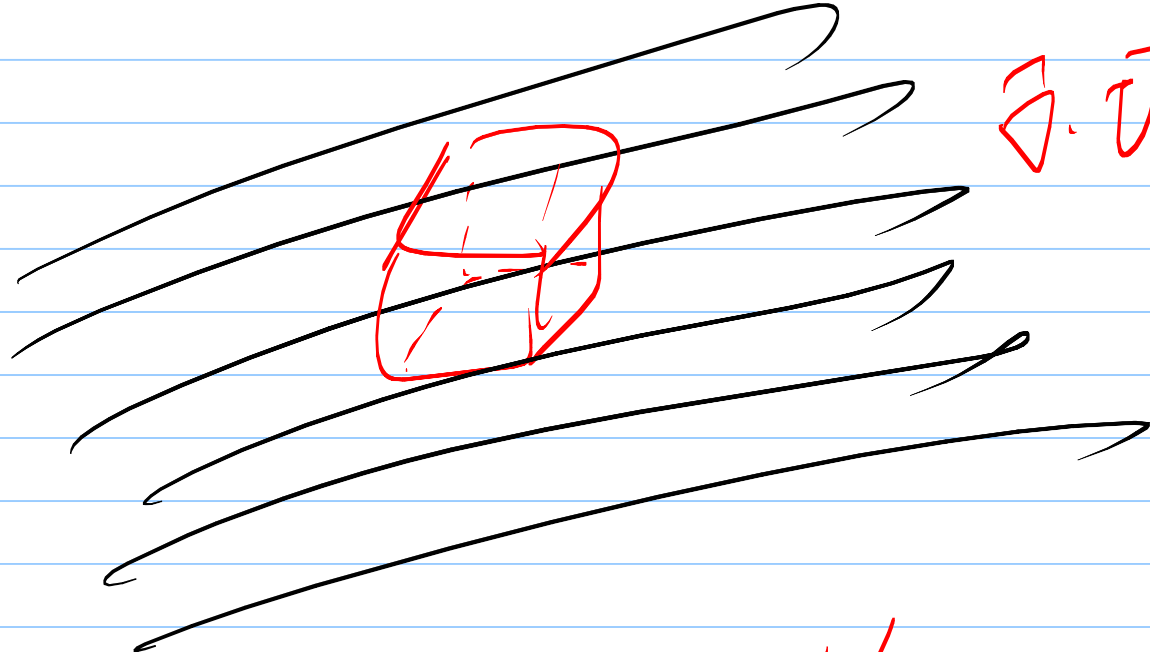
$$\approx \left(\frac{\partial U}{\partial y}\right) \cdot dy\, dxdz$$

$$= \left(\frac{\partial U}{\partial y}\right) dxdydz.$$

$$\left(\frac{\partial U_x}{\partial x}\right) dx\,dy\,dz + \left(\frac{\partial U_y}{\partial y}\right) dx\,dy\,dz + \left(\frac{\partial U_z}{\partial z}\right) \cdot dx\,dy\,dz$$

$$= dx\,dy\,dz \cdot \left(\vec{\nabla} \cdot \vec{U}\right) \qquad dU$$





$\vec{\nabla} \cdot \vec{U} = 0$

Source

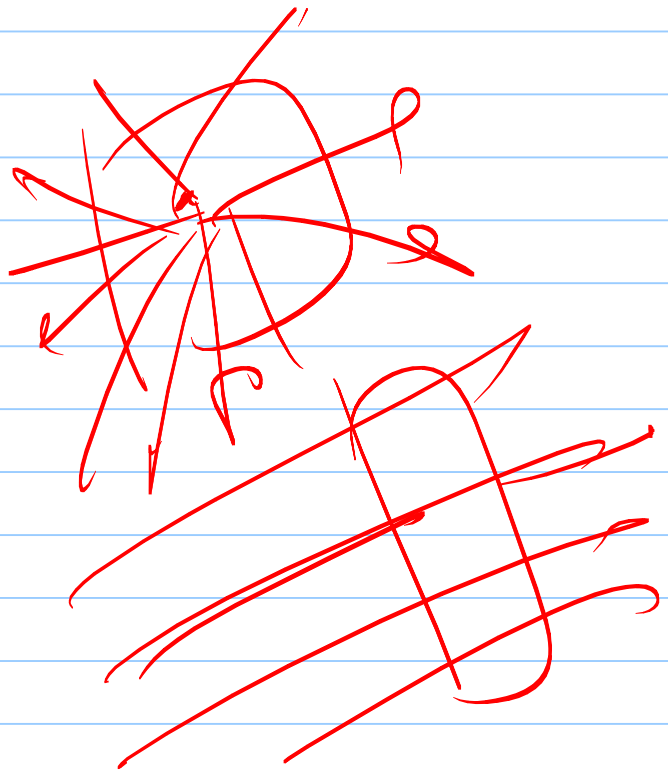
U

dV

$$\frac{\partial \rho}{\partial t} dV = -\nabla \cdot \vec{V} dV + \psi dV.$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{V} = \psi}$$

$\psi = 0$: [illegible]



$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{V} = 0.$$

Continuity eq.

$$\frac{\partial \rho}{\partial t} = 0. \quad \psi \neq 0.$$

$$\boxed{\vec{\nabla}\cdot\vec{v} = \psi} \neq 0$$

$dx\,dz\,dy$

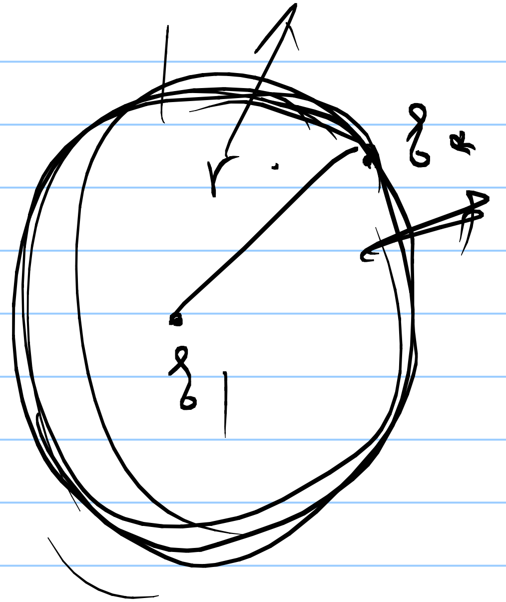
$$\int_V \vec{\nabla}\cdot\vec{v}\,dV = \int_A \vec{v}\cdot d\vec{A}$$

$dx\,dy$

Gauss' Law.

$$\vec{F}_c = \frac{1}{4\pi\epsilon_0} q_1 q_2 \frac{\vec{r}}{r^3} = \frac{1}{r^2}\hat{r}$$

$$\int \underline{\vec{\nabla}\cdot\vec{F}}\, dV = \int_A \vec{F}\cdot \underline{d\vec{A}} = \frac{q_1 q_2}{4\pi\epsilon_0}\int \frac{1}{\cancel{r^2}}\cdot \boxed{\cancel{r^2}\sin\theta\, d\theta\, d\phi} = \cancel{4\pi}$$



$\vec{F} \sim \hat{r}$ $d\vec{A} \sim \hat{r}$

$\hat{F}\cdot d\vec{A} = |\vec{F}|\cdot|d\vec{A}|$

$$dA = dx\,dy \to (r\,d\theta)(r\sin\theta\, d\phi)$$
$$= r^2 \sin\theta\, d\theta\, d\phi$$

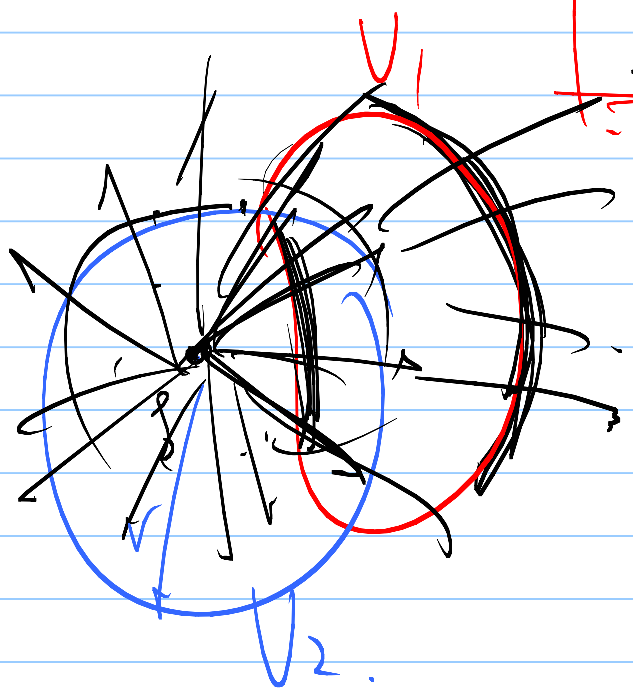
$$\int \vec{\nabla}\cdot\vec{F}\, dV = \frac{q_1 q_2}{\cancel{4\pi}\epsilon_0}\cdot\cancel{4\pi} = \frac{q_1 q_2}{\epsilon_0} \qquad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}$$

$$\boxed{\cancel{\int} \vec{\nabla}\cdot\vec{E}\,\cancel{dV} = \frac{q_1}{\epsilon_0}} = \frac{1}{\epsilon_0}\cancel{\int}\rho\,\cancel{dV}.$$

ρ = 전하밀도.

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$$

Maxwell #1.



$$\int_{V_1} \vec{\nabla}\cdot\vec{E}\,dV =$$

$$\vec{\nabla}\cdot\frac{\vec{r}}{r^3} = 0$$

$$\vec{\nabla}\cdot\frac{\hat{r}}{r^2} = 0.$$

$$\vec{F}_N = G m_1 m_2 \frac{\hat{r}}{r^2}.$$

$$\int_{V_2} \vec{\nabla} \cdot \vec{E}\, dV = \int \frac{\rho}{\epsilon_0}\, dV = \frac{Q}{\epsilon_0}$$

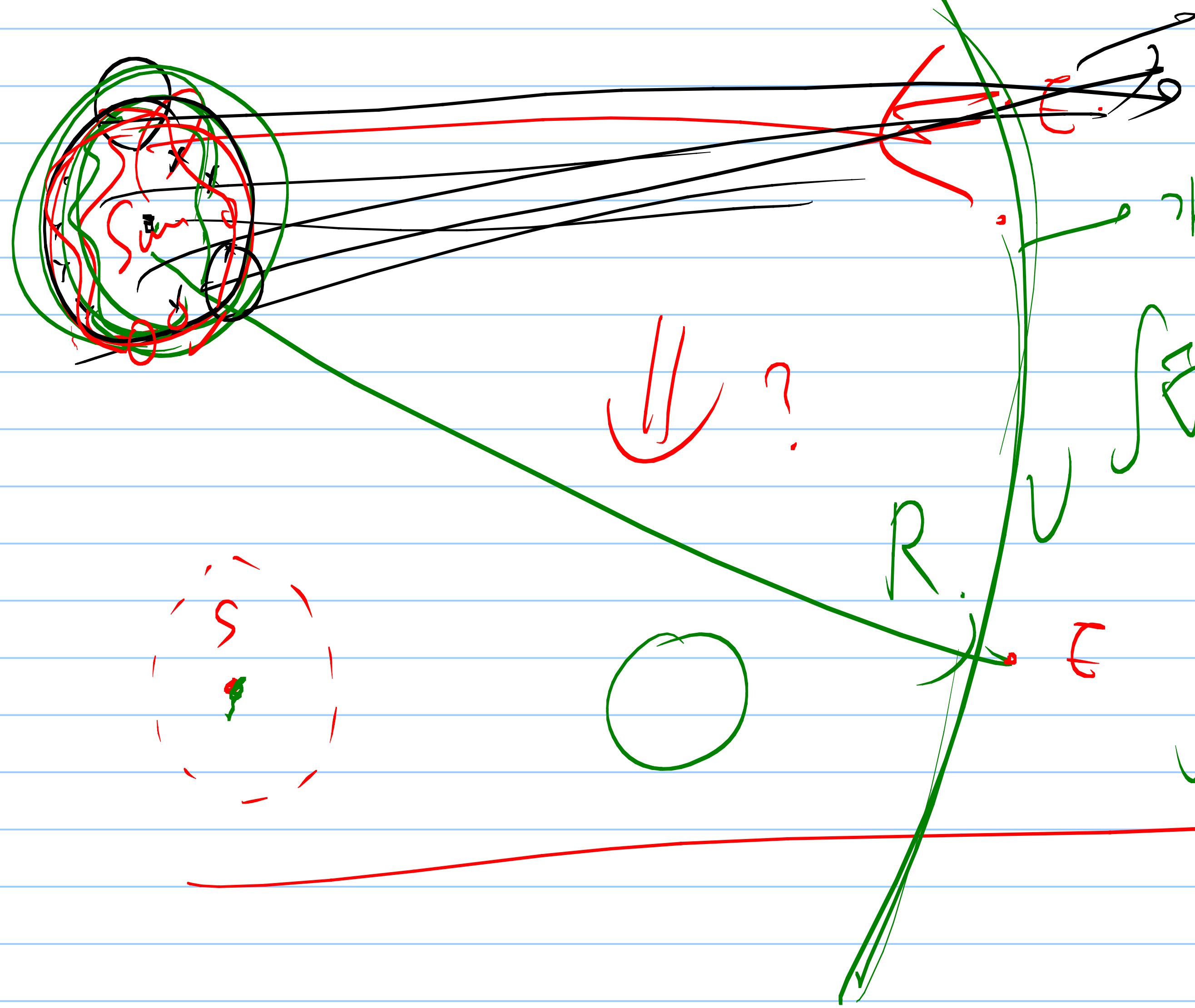
$$=$$

$$\int_{\partial V_2} \overset{\hat{r}}{\vec{E}} \cdot \overset{\hat{n}}{d\vec{A}} = E \cdot 4\pi r^2 = \frac{\rho}{\epsilon_0}$$

$$\to \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2}}$$

$$F = \frac{G m_1 m_2}{r^2}.$$

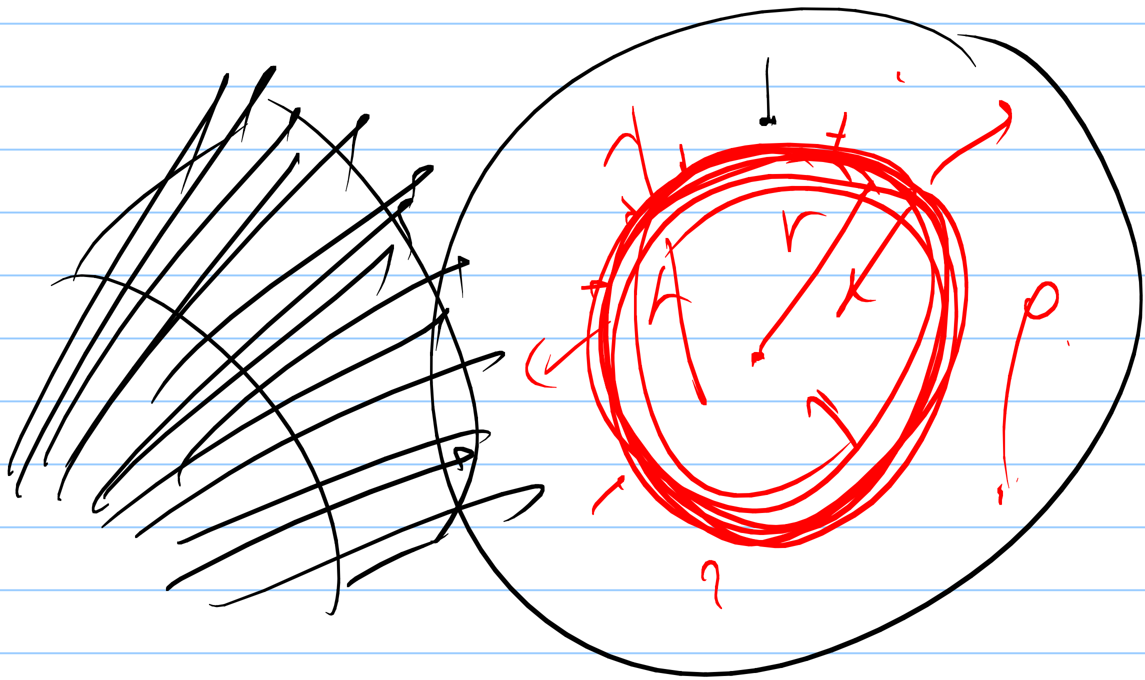
$m_1 \quad m_2$



$$\int \nabla \cdot \vec{F}\, dV = \sim M.$$

$$\Downarrow$$

$$\int \vec{F} \cdot d\vec{A} = F \cdot 4\pi r^2.$$



Su..

$$\int \vec{\nabla}\cdot\vec{E}\, dV = M$$

flux

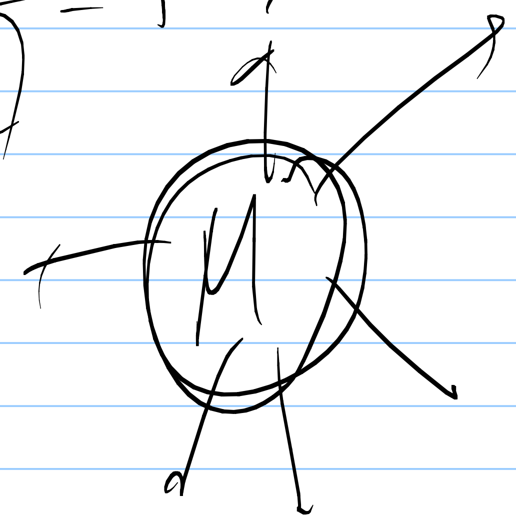
$$\int \vec{\nabla}\cdot\vec{U}\, dV = \int_{\partial V} \vec{U}\cdot d\vec{A}$$

$$\int \vec{\nabla} \times \vec{U} \cdot d\vec{A} = \int U \, dv$$

중력상수.

질량밀도

$$\frac{\vec{F}}{m} = \vec{f} \rightarrow$$



$$\vec{\nabla} \cdot \vec{f} = 4\pi G \rho$$

$$\underline{\int \vec{\nabla} \cdot \vec{f} \, dV} = \int 4\pi \rho G \, dV$$

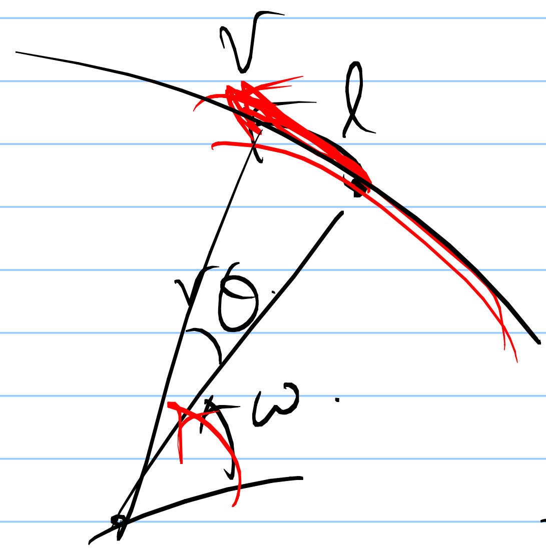
$$= \underline{M 4\pi G}.$$

$$\| $$

$$\int \vec{f} \cdot d\vec{A} = f \cdot \underline{4\pi r^2}.$$

$$f = \frac{GM}{r^2}$$

$$\boxed{\vec{\nabla} \times \vec{v}}$$



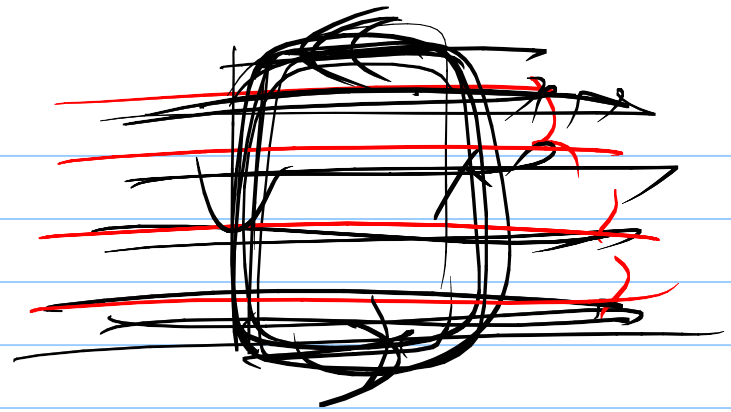
$$v = r\omega$$

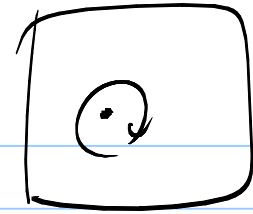
$$v = \frac{dl}{dt} = \frac{d}{dt}(r\theta) = r\frac{d}{dt}\theta = r\omega$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{\omega} = \frac{1}{2}(\vec{\nabla} \times \vec{v})$$

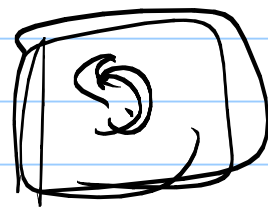
!!



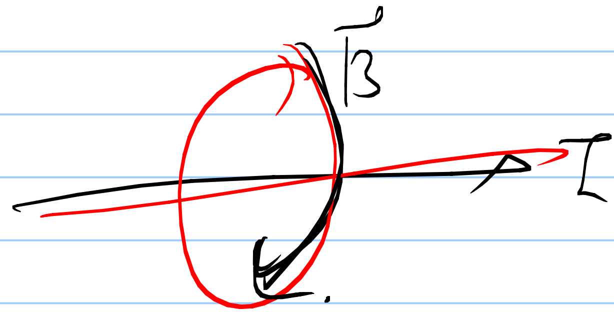


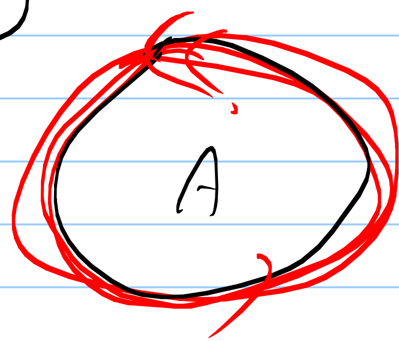
$$\int_A \underline{\underline{\vec{\nabla} \times \vec{U} \cdot d\vec{A}}} \; \underline{\underline{dx\,dy}} = \int_{\partial A} \underline{\vec{U} \cdot d\vec{r}}$$

Stokes theorem.

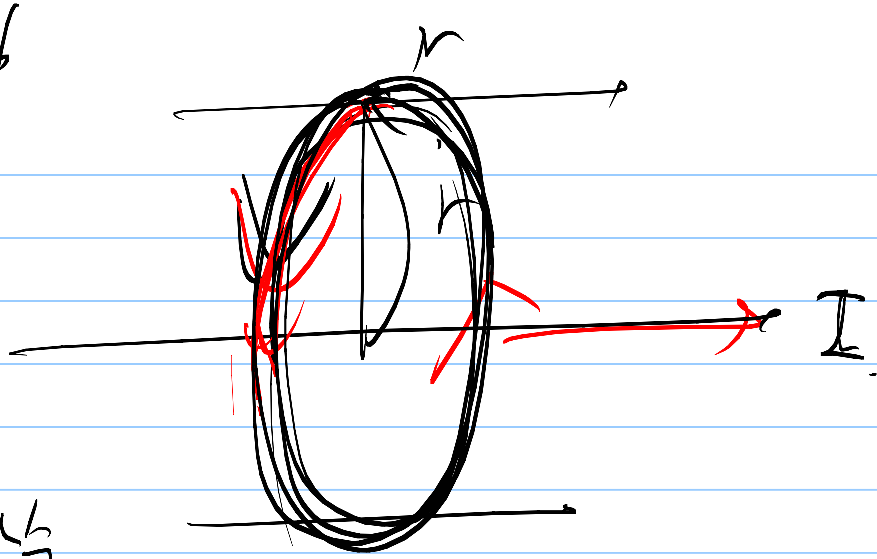


$$\boxed{\vec{B} = \mu_0 \vec{H}}$$





$$\oint \underline{\underline{H}} \cdot dr = I$$



상수.

$$\oint \underline{\underline{H}} \cdot \underline{\underline{dr}} = \boxed{H \cdot 2\pi r} = I$$

$$H = \frac{I}{2\pi r}$$

단위면적당 전류.

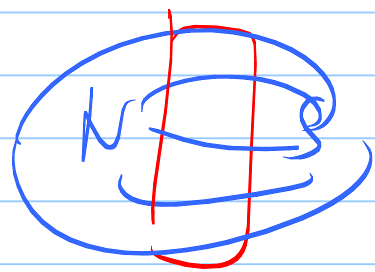
$$I = \int \vec{J} \cdot d\bar{A}$$

current density.

$$\oint \vec{H} \cdot \underline{\underline{d\vec{r}}} = I = \int \vec{J} \cdot d\vec{A}.$$

$$\int_A \vec{\nabla} \times \vec{H} \cdot d\vec{A} = \int_A \vec{J} \cdot d\vec{A}.$$

$$\underline{\underline{\vec{\nabla} \times \vec{H}}} = \vec{J}.$$



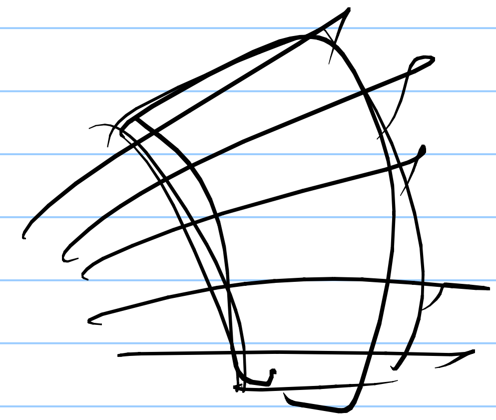
Maxwell's Eqs.

쿨롱법칙. $\leftarrow$ $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$

No monopole. $\leftarrow$ $\vec{\nabla}\cdot\vec{B} = 0$

Ampere's law $\leftarrow$ $\vec{\nabla}\times\vec{B} = \epsilon_0\mu_0 \dfrac{\partial \vec{E}}{\partial t} + \mu_0\vec{J}$

Faraday's Law. $\leftarrow$ $\int \vec{\nabla}\times\vec{E}\,d\vec{A} = -\int \dfrac{\partial \vec{B}}{\partial t}\,dA.$



$$\boxed{\begin{aligned} \int_V \vec{\nabla}\cdot\vec{U}\,dV &= \int_{\underline{\partial V}} \vec{U}\cdot d\vec{A} \\ \int_A \vec{\nabla}\times\vec{U}\cdot d\vec{A} &= \oint_{\partial A} \vec{U}\cdot d\vec{r} \end{aligned}}$$

$$\underline{\vec{\nabla}}.$$

Taylor expansion.

$$f(x) = \sum a_n x^n . = \sum f^{(n)}(0)\, x^n .$$

$$= \sum f^{(n)}(x_0)\,(x-x_0)^n .$$

Flourier expansion.

$$f(x) = a_0 \sum \left(a_n \sin nx + b_m \cos mx \right)$$

$$\int_{0\,(-\pi)}^{2\pi\,(\pi)} \sin nx \cos mx \, dx = 0. \qquad m, n \in \mathbb{Z}$$

$$\int_0^{2\pi} \sin nx \sin mx \, dx \neq 0 \text{ if } \boxed{m = n}$$

$$\cos 2x = 1 - 2\sin^2 x. \qquad \cancel{\int_0^{2\pi} \sin x \sin 2x \, dx = 0} \qquad \cancel{\cos(x) - \cos(x\ldots)}$$

$$\int_0^{2\pi} \sin^2 x \, dx = \int_0^{2\pi} \frac{1 - \cos 2x}{2} \, dx$$

$$= \frac{1}{2} \cdot 2\pi + 0.$$

$$\frac{1}{2\pi}\int_0^{2\pi} \sin^2 x\,dx = \frac{1}{2}.$$

orthogonal.

$$\frac{1}{2\pi}\int_0^{2\pi} \sin mx \sin nx\,dx = \frac{1}{2}\delta_{mn}. \qquad \delta_{mn} = \begin{cases} 1 & \text{if } m=n \\ 0 & \text{if } m \neq n. \end{cases}$$

$$\vec{x}\cdot\vec{y} = 0.$$

$$f(x) = a_0 + \sum\left(a_m \sin mx + b_n \cos nx\right)$$

$$\boxed{\frac{1}{2\pi}\int_0^{2\pi} f(x)\sin kx\,dx} = 0 + \sum_m a_m \frac{1}{2\pi}\int_0^{2\pi} \sin mx \sin kx\,dx$$

$$= 0 + \sum_{m=0}^{\infty} a_m \cdot \frac{1}{2}\delta_{mn}$$

$$= a_n \cdot \frac{1}{2}$$

$$\therefore\ a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$(x, y, z) \to (r, \theta, \phi)$$

$$dx, dy, dz \to dr, r\,d\theta, r\sin\theta\,d\phi$$

$$\to (\xi_1, \xi_2, \xi_3)$$

$h_1 d\xi_1$

$$d\vec{r} \to h_1 d\xi_1 \hat{\xi}_1 + h_2 d\xi_2 \hat{\xi}_2 + h_3 d\xi_3 \hat{\xi}_3$$

$$ds^2 = (h_1 d\xi_1)^2 + (h_2 d\xi_2)^2 + (h_3 d\xi_3)^2$$

$$\vec{\nabla}\phi = \sum_i \frac{\partial \phi}{\partial x_i} = \sum_{i=1}^{3} \frac{\partial \phi}{h_i \partial \xi_i}$$

$h_1 = 1, \quad h_2 = r, \quad h_3 = r\sin\theta$

$1\,dr \qquad r\,d\theta \qquad r\sin\theta\,d\phi$

$h_1 \qquad h_2 \qquad h_3$

계량텐서 (metric tensor)

$g_{ij} \rightarrow g_{\mu\nu}$

$$ds^2 = dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2.$$