노트 제목 2009-03-07

$$\theta = \frac{l \in 길이\ (\cancel{m})}{r \in 길이\ (\cancel{m})}\ (rad)$$ <u>dimensioless</u>.

$$v = \frac{s \in m}{t \in s} = m/s.$$ dimension

$\pi = 180^\circ$

$1^\circ = \frac{\pi}{180}$

$=$

$$\underset{0}{\sin 2\pi} \neq 2\pi \qquad 2\pi \gg 1$$

$$\sin 1^\circ = \sin \frac{\pi}{180} \approx \frac{\pi}{180}.$$

$$\sin 0.1 \approx 0.1.$$

$360^\circ = 2\pi = 6.28$

$\frac{l}{r}$ (rad)

$\boxed{\sin x} \to x$

$\sin x^\circ$

평균변화율 / 순간변화율 = 미분계수.

$\frac{\Delta y}{\Delta x}$ $\quad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}.$

$$\boxed{f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$\left(cf(x) \pm g(x)\right)' = cf'(x) \pm g'(x)$$ 선형성. (linear)

$$\frac{d}{dx}\left(cf \pm g\right) = cf' \pm g'$$

operation (연산작용)

$$\left[f(x)g(x)\right]' = f'g + fg'$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}.$$

$$\boxed{\left(\frac{1}{g(x)}\right)' = \frac{-g'(x)}{g^2(x)}}$$

$$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x)\cdot\frac{1}{g(x)}\right)'$$

$$= f'\cdot\frac{1}{g(x)} + f\cdot\left(\frac{1}{g}\right)'$$

$$= \frac{f'}{g} - \frac{fg'}{g^2} = \boxed{\frac{f'g - fg'}{g^2}}$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)'\cos x - \sin x\cdot(\cos x)'}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\lim_{b\to a}\frac{f(b)-f(a)}{\boxed{b-a}}$$

$$[f(g(x))]'$$

$$= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

$$= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \times \frac{g(x+h) - g(x)}{h}$$

$$= f'(u) \cdot g'(x) = f'(g(x)) \cdot g'(x)$$

$$[f(\overset{u}{g(x)})]'$$

$f'(u) \cdot u'$

$(\sin x)' = \cos x.$

$[\sin(2x^2+3x)]' = ?$

$\| \; u = g(x) = 2x^2+3x.$

$y = f(u) = \sin u.$

$\Downarrow$

$\sin(2x^2+3x) = f(g(x))$

$g(x) = u.$

$$\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

$u = g(x)$

$$\sin(2x^2+3x)$$

$u = 2x^2+3x$

$$\left[\sin(2x^2+3x)\right]' = (\cos u) \cdot (4x+3)$$

$$= \left[\cos(2x^2+3x)\right] \cdot 4x+3$$

$$= (4x+3)\cos(2x^2+3x)$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\left(\frac{1}{g(x)}\right)' = -\frac{1}{u^2} \cdot u' = -\frac{g'(x)}{[g(x)]^2}.$$

$g(x) = u$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\sqrt{x^2+\sin x} = \frac{1}{2\sqrt{(\quad)}}(\ \cdot\ )'$$

$$= \frac{1}{2\sqrt{x^2+\sin x}} \cdot (2x+\cos x).$$

음함수 (implicit function) ↔ 양함수 (explicit ftn),

$y - f(x) = 0$.　　　$y = f(x)$

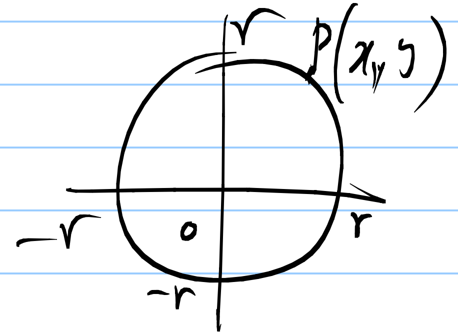
$y - 2x^2 = 0$　　　$y = 2x^2$

$xy = 1$　　　$y = \frac{1}{x}$

원 : $\boxed{x^2 + y^2 = r^2}$.　　　$y = \pm\sqrt{\quad}$

$\sin x + \sin y = 1.$ $\qquad y = \sin x.$

$xy^2 + x^2y + \sin(xy) = 0$



$\boxed{x^2 + y^2 = r^2.}$

원점에서 거리가 r인 점들의 집합.

$O(0,0) \qquad P(x,y)$

$y' =$

$\overline{OP}^2 = r^2 = (x-0)^2 + (y-0)^2$

$\frac{dy}{dx} = ?.$

$y =$

$\frac{d}{dx} y.$

$$x^2+y^2=r^2$$

$$\frac{d}{dx}\left(x^2+y^2\right) = \frac{d}{dx}r^2 = 0.$$

$$\frac{d}{dx}(y\cdot y) = y'y + y\cdot y'$$
$$= 2y\cdot y'$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = 0.$$

$$2x + \left(\frac{d}{dy}\cdot y^2\right)\cdot\frac{dy}{dx} = \boxed{2x+2y\cdot y'} = 0.$$

$$\therefore \frac{d}{dx}y = y' = -\frac{x}{y}.$$

$$\frac{d}{dx}.$$

d : 미분, 극소변화량 $(\Delta x \to 0)$

$x^2 + y^2 = r^2$.

$d(x^2 + y^2) = d(r^2) = 0$.

$\dfrac{d}{dx} x^2 = 2x$.

$\Rightarrow dx^2 = 2x\,dx$.

$dx^2 + dy^2 = 0$.

$2x \cdot dx + 2y\,dy = 0$.

$\dfrac{dy}{dx}$

$2x + 2y \dfrac{dy}{dx} = 0$. $\Rightarrow y' = \dfrac{dy}{dx} = -\dfrac{x}{y}$.

$$\frac{d}{dx} f(x) = f'(x) \Rightarrow \boxed{df(x) = f'(x) \cdot dx}$$

$$\int df(x) = \int f'(x)\, dx$$

$$f(x) =$$

$$y = g(x) = f^{-1}(x) \Rightarrow \boxed{f(g(x)) = x}$$

$$\rightarrow \left[f(g(x))\right]' = 1$$

$$f'(y) \cdot g'(x) = 1$$

$$\Rightarrow g'(x) = \frac{1}{f'(y)} = \frac{1}{f'(g(x))}$$

$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x.$$

$$[f(g(x))]' = f'(g(x)) \cdot g'(x).$$

지수·로그함수

$$y = a^{x} \Rightarrow x = \log_{a} y. \quad y > 0. \quad (a \neq 1, a > 0)$$

$$(\log_a x)' = \lim_{h \to 0} \frac{\log_a(x+h) - \log_a x}{h}.$$

$$= \lim_{h \to 0} \frac{1}{h} \log_a \left(\frac{x+h}{x}\right).$$

$$\log_a x - \log_a y = \log_a \frac{x}{y}.$$

$$= \lim_{h \to 0} \frac{1}{h} \log_a \left(1 + \frac{h}{x}\right).$$

$$\log_a x^n = n \log_a x.$$

$$e = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$$

$$= \lim_{h\to 0}\log_a\left(1+\frac{h}{x}\right)^{\frac{1}{h}}$$

$$= \lim_{h\to 0}\log_a\left[\left(1+\frac{h}{x}\right)^{\frac{1}{\frac{h}{x}}}\right]^{\frac{1}{x}}$$

$$\log_a x = \frac{1}{\log_x a}$$

$$= \log_a e^{\frac{1}{x}}$$

$$= \frac{1}{x}\log_a e = \frac{1}{x}\,\frac{1}{\log_e a}$$

$$\therefore (\log_a x)' = \frac{1}{x}\,\frac{1}{\log_e a}$$

$a = e$:

$$\boxed{(\log x)' = \frac{1}{x}}$$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C. \quad \text{for } n \neq -1.$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \cdot dx = ? \; \log x + C.$$

$$y = e^x.$$

$$(e^x)' = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = e^x \boxed{\lim_{h \to 0} \frac{e^h - 1}{h}} = e^x.$$

$e^{x+h} = e^x \cdot e^h.$

"1"

$$\boxed{\frac{d}{dx} e^x = e^x.}$$

$$\boxed{e^{\log_e x} = x.}$$

$e^t = x.$

$t = \log_e x$

$$(a^x)' = \left(e^{\log a^x}\right)' = \left(e^{\boxed{x \log a}}\right)' = e^u \cdot \log a.$$

"u"

"a^x"

$e^x = a^x \log_e a$ $\quad a \neq e$

$\boxed{(e^x)' = e^x}$

$\boxed{(\log_e x)' = \frac{1}{x}}$

$(a^x)' = a^x \cdot \log a$

$\boxed{(\log_a x)' = \frac{1}{x} \cdot \frac{1}{\log a}}$

$\log_a b = \frac{\log_c b}{\log_c a}$

$$= \left(\frac{\log_e x}{\log_e a}\right)' = \frac{1}{\log a} \cdot \frac{1}{x}.$$

$$\boxed{(e^x)' = e^x};$$

$$(\log x)' = ?$$

$$\left(e^{\log x}\right) = x. =$$

$$f\left(f^{-1}(x)\right) = x.$$

$$\left(e^{\overset{u}{\log x}}\right)' = e^u \cdot \boxed{(\log x)'} = x' = 1.$$

$$\therefore (\log x)' = \frac{1}{e^u} = \frac{1}{e^{\log x}} = \frac{1}{x}!$$

$(x^n)' = n x^{n-1}$.

$$\underline{(x^n)'} = \left(e^{\log x^n}\right)' = \left(e^{\underline{n \log x}}\right)'$$

$$= u.$$

$$= e^u$$

$$= e^u \cdot n \cdot \frac{1}{x}.$$

$$= x^n \cdot n \frac{1}{x} = \underline{n x^{n-1}}.$$

for $n \in \mathbb{R}$.

$\log_e x = \underline{\log x} = \underline{\ln} \, x$.

$$\frac{d}{dx}\log(x^3+1) = \frac{1}{u}\cdot u' = \frac{2x^2}{x^3+1}.$$

"u."

$$y = x^{\sin x}. \qquad e^{x} \qquad x^{3}$$

$$\log y = \log x^{\sin x}$$

$$= (\sin x)\cdot \log x.$$

$$\frac{d}{dx}\log y = \frac{d}{dx}\left[(\sin x)(\log x)\right] = (\cos x)\cdot \log x + \frac{\sin x}{x}.$$

$$= \frac{d}{dy}\log y \cdot \frac{dy}{dx} = \frac{1}{y}\cdot\frac{dy}{dx} = (\cos x)\log x + \frac{\sin x}{x}$$

$$\therefore \frac{dy}{dx} = y\left[(\cos x)\log x + \frac{\sin x}{x}\right]$$

$$= x^{\sin x}\left[\qquad + \qquad\right]$$

$$(e^x)' = e^x. \Rightarrow \frac{d}{dx}e^x = e^x.$$

$$\frac{d}{dx}f(x) = f(x). \Rightarrow d\,f(x) = f(x)\,dx.$$

$$\boxed{\frac{df(x)}{f(x)}} = dx.$$

$$\boxed{(\log x)' = \frac{1}{x}}. \qquad \frac{d}{dx}\log x = \frac{1}{x}. \Rightarrow \boxed{d\log x = \frac{dx}{x}}.$$

$$\frac{df(x)}{f(x)} = d\left[\log f(x)\right] = dx.$$

미분방정식.

$$\int 1 \cdot d\left[\log f(x)\right] = \int 1\, dx = x.$$

$$\frac{3}{3!} = \frac{\cancel{3}}{\cancel{3}\times 2\times 1}$$

$$\boxed{\log f(x) = x.} \rightarrow f(x) = e^{x}.$$

$$e^{x} = 1 + x + \frac{1}{2!}x^{2} + \frac{1}{3!}x^{3} + \cdots + \frac{1}{n!}x^{n} + \frac{x^{n+1}}{(n+1)!} + \cdots$$

$$(e^x)' = 0 + \underline{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{1}{(n-1)!}x^{n-1} + \frac{1}{n!}x^n + \cdots}$$

$$= \underline{e^x} = \sum_{k=0}^{\infty} \frac{x^k}{k!}.$$

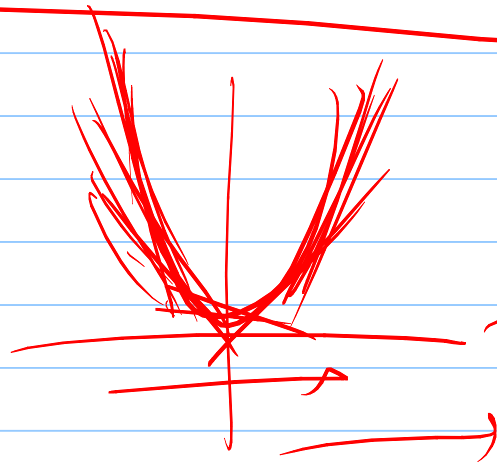
$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x.$$

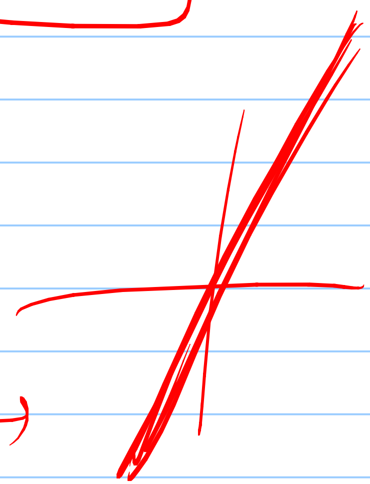
$$e = \lim_{x \to 0} (1+x)^{\frac{1}{x}}$$

$$(e^x)' = e^x, \quad (\log x)' = \frac{1}{x}.$$

$$(x^2)' = 2x.$$

$$(e^x)' = e^x$$

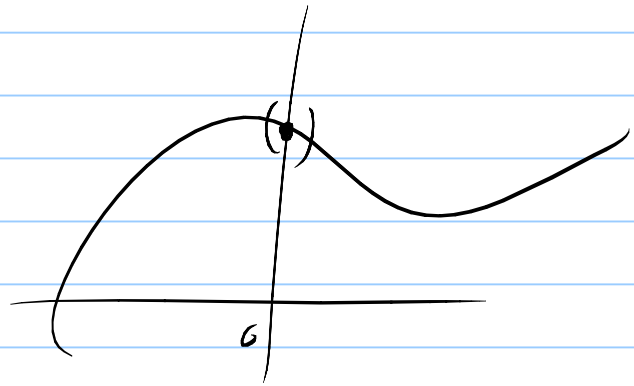




$$f(x) = \sum (\text{다항함수}) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots$$

s.c.e.

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$



$$(e^x)^{(n)}_{x=0} = (e^0)\ 1$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

$$(\sin x)'_{x=0} = \cos x\big|_0 = 1$$

$$(\sin x)''_{x=0} = -\sin x\big|_0 = 0$$

$$(\sin x)'''_0 = -\cos x\big|_0 = -1$$

$$\therefore\ \boxed{\sin x = 0 + 1 \cdot x} + 0 - \frac{1}{3!}x^3$$

odd $+\frac{x^5}{5!}+\cdots$

$(\cos x)'_0 = -\sin x|_0 = 0$.

$(\cos x)''_0 = -\cos x|_0 = \underline{\underline{-1}}$

$(\cos x)'''_0 = \sin|_0 = \underline{0}$

$(\cos x)^{(4)}_0 = \cos x|_0 = \underline{+1}$

$\cos x = 1 + 0 - \frac{1}{2!}x^2 + 0 + \frac{x^4}{4!}$

$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\cdots$

$i^2 = -1$

e^{ix}

$$\boxed{\begin{aligned}\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\end{aligned}}$$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$e^{ix} = 1 + ix - \frac{x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + i\left(x - \frac{x^3}{3!} + \cdots\right)$$

$$= \cos x + i\sin x$$

$$\therefore \boxed{e^{ix} = \cos x + i\sin x.}$$

$$f(x) = \sum(\text{다항함수})$$

$$= \sum(\sin x \text{ or } \cos x)$$

Fourier exp.