비율이 일정.

(무한) 등비급수 합.

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots = \infty.$

$1+\underbrace{\frac{1}{2}}_{\frac{1}{2}}+\underbrace{\frac{1}{4}}_{\frac{1}{2}}+\underbrace{\frac{1}{8}}_{\frac{1}{2}}+\cdots = 2.$

공비 (common rate) r.

등비수열.

$1,\ 2,\ 4,\ 8,\ 16,\ \cdots$ 수열.

(공비 2, 2, 2, 2)

$ar^0 \quad ar^1 \quad ar^2 \quad ar^3 \quad ar^4 \quad \cdots \quad |$ $\boxed{ar^{\boxed{n-1}}}$

초항

1항 2항 3 4 5. n번째 항

$\times r.$

$$S = \overbrace{a + ar + ar^2 + \cdots + ar^{n-1}}^{n\text{개}} = ?$$

$(n-1)$개

$$-)\ rS = ar + ar^2 + ar^3 + \cdots + ar^n \qquad (r \neq 1)$$

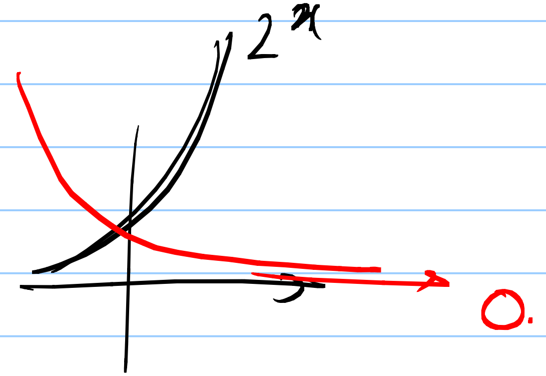
$$(1-r)S = a - ar^n = a(1-r^n)$$

n → 항의 개수.

$$\therefore S = \frac{a(1-r^n)}{1-r} \quad (r \neq 1) \qquad (S = na \text{ for } r = 1.)$$

등비수열의 합.

$$\lim_{n\to\infty} S = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r}$$



$$\lim_{n\to\infty} r^n = \begin{cases} \infty & \underline{r>1} \\ 1 & r=1 \\ 0 & \underline{-1<r<1} \quad (\underline{|r|<1}) \\ 진동 & r=-1 \\ 진동.\ 발산. & r<-1 \end{cases}$$

2^n

$\left(\frac{1}{2}\right)^n$.

$(-1)^n = \begin{cases} 1 & \text{for } n \text{ even} \\ -1 & \text{for } n \text{ odd.} \end{cases}$

$(-2)^n$.

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \boxed{\frac{a(1-r^n)}{1-r}} = \frac{a}{1-r} \text{ for } |r|<1.$$

초항

공비

$$= \lim_{n\to\infty} \sum_{k=1}^{n} ar^{k-1} = (a+ar+ar^2+\cdots+ar^{n-1})$$

$$= \sum_{k=1}^{\infty} \underline{ar^{k-1}} = \frac{a}{1-r} \text{ for } \underline{|r|<1}.$$

$$\underbrace{1+\frac{1}{2}}_{\frac{1}{2}}\underbrace{+\frac{1}{4}}_{\frac{1}{2}}\underbrace{+\frac{1}{8}}_{\frac{1}{2}}+\cdots = 1 \times \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2.$$

$$\underline{0.9999\cdots} = 0.9 + 0.09 + 0.009 + \cdots$$

$$= 0.9 \times \frac{1}{1-\frac{1}{10}} = \frac{0.9}{0.9} = \underline{1}.$$

$$\checkmark \left(\sum_{n=0}^{\infty} e^{-nh\nu/kT} \right) = \left(1 \cdot \frac{1}{\underline{1-e^{-nh\nu/kT}}} \right) = \frac{a}{1-r}$$

순열 / 조합.

순열 : Permutation. = 줄 세우는 경우의 수.

A B C

ABC, ACB, BAC, BCA
CAB, CBA 6개.

$\boxed{3} \times \boxed{2} \times \boxed{1}$ = 6 개 = 3! (계승, factorial)

(n개를 줄세우는 경우의 수) = $n \times (n-1) \times \cdots \times 2 \times 1 = n!$

100000⋯0

$x^n < 2^x < x!$

$0! = 1,\quad 1! = 1,\quad 2! = 2,\quad 3! = 6,\quad 4! = 24$

$5! = 120.$

$6! = 720.$

~~5명 중 2명~~

$\cancel{\boxed{5} \times \boxed{4}} = 20 = {}_5P_2.$

${}_nP_r$: n명 중 r명을 줄세우는 경우의 수.

순서가 중요.

AB, BA

$= \underbrace{\boxed{n} \times \boxed{n-1} \times \cdots \times \boxed{\ }}_{r\text{개}}$

$n-0 \quad n-1 \quad n-2 \quad n-(r-1)$

Permutation $= \dfrac{\overbrace{n\times(n-1)\times\cdots\times(n-r+1)(n-r)(n-r-1)\times\cdots\times 2\times 1}^{n!}}{\underbrace{(n-r)(n-r-1)\times\cdots\times 2\times 1}_{=(n-r)!}}$

$= \underbrace{n\times(n-1)\times\cdots\times(n-r+1)}_{r개}$

$\therefore\ {}_nP_r = \dfrac{n!}{(n-r)!} = n\times\cdots\times(n-r+1)$

${}_5P_2 = \dfrac{5!}{(5-2)!} = \dfrac{5!}{3!} = 5\times 4 = 20$ 순서를 따짐

조합 (Combination)

32개국, 8조.

1조 : 4팀 → 16강 ↑

A B C D

AB, BC, CD, DA, AC, AD

6가지.

$\frac{AB}{BA}$ 순서를 안 따짐

$_nC_r$: n개 중 r개를 선택하는 경우의 수. 조합.

$_nP_r$ = (n개 중 r개를 줄세우는 경우의 수)

= (n개 중에서 r개를 선택하는 경우의 수) × (r개를 줄세우는 경우의 수)

$= {}_nC_r \times r!$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$\therefore\ {}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{(n-r)!\,r!} =$$

$${}_nC_{n-r} = \frac{n!}{(n-r)!\,(n-(n-r))!} = \frac{n!}{(n-r)!\,r!}$$

5개 중 3개를 선택 = 3개 선택, 2개 버림.

= 2개 선택, 3개 버림.

$${}_5C_3 = {}_5C_2.$$

= 3개 / 2개

$${}_nC_r = {}_nC_{n-r}.$$

= 2개와 3개.

$${}_5C_2 = \frac{5!}{2!\,3!} = \frac{{}_5P_2}{2!} = \frac{5\times 4}{2!} = 10\text{ 가지.}$$

$${}_4C_2 = \frac{4\times 3}{2!} = 6$$ 가지.

$${}_{10}C_9 = \frac{10\times 9\times\cdots 2}{9!} = {}_{10}C_{10-9} = {}_{10}C_1$$

$$= \frac{10\times 1}{1} = 10.$$

In general

$$\boxed{\begin{aligned} &{}_nC_0 = 1 = {}_nC_n = 1. \\ &{}_nC_1 = n, \quad {}_nC_2 = \frac{n(n-1)}{2!} \end{aligned}}$$

이항정리 $(a+b)^n =$

$$(a+b)^3 = (a+b)(a+b)(a+b)$$

$$= 1\,a^3 + 3a^2b + 3ab^2 + b^3$$

$$_3C_3 = {}_3C_0$$

$$_3C_2 = {}_3C_1$$

$$_3C_1 = {}_3C_2$$

$$(a+b)^n = a^n + {}_nC_1 a^{n-1}b + {}_nC_2 \cdots + {}_nC_2 a^2 b^{n-2} + {}_nC_1 a b^{n-1} + \overset{{}_nC_0}{b^n}$$

$$= \boxed{\sum_{r=0}^{n} {}_nC_r\, a^r b^{n-r}} = \sum_{r=0}^{n} {}_nC_{n-r}\, a^r b^{n-r}$$

$$= \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

$$(x^n)' = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^n} + {}_nC_1 x^{n-1} \cdot \cancel{h} + {}_nC_2 x^{n-2} h^{\cancel{2}} + \cdots + {}_nC_n h^n - \cancel{x^n}}{\cancel{h}} \quad 0.$$

$$= \underline{\underline{{}_nC_1}}\, x^{n-1} = n x^{n-1}.$$

6/45 $\quad \dfrac{1}{{}_{45}C_6} = \dfrac{1}{8,160.-}$

$${}_{45}C_6 = \frac{\cancel{45}^{15} \times 44 \times 43 \times \cancel{42}^{7} \times 41 \times \cancel{40}}{\cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 15 \times 44 \times 43 \times 7 \times 41$$

$$\begin{array}{r} 44 \\ \times\ 43 \\ \hline 132 \\ 176\ \ \\ \hline 1892 \end{array} \qquad \begin{array}{r} 287 \\ \times\ 15 \\ \hline 1435 \\ 287\ \ \\ \hline 4305 \end{array}$$

$$\begin{array}{r} 4305 \\ \times\ 1892 \\ \hline 8610 \\ 38745\ \ \\ 34440\ \ \ \ \\ 4305\ \ \ \ \ \ \\ \hline 8,165,160 \end{array}$$

- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

  $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

- $\underline{\lim_{x\to 0}} \dfrac{\sin x}{x} = 1$, $\quad e = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$

  $\Rightarrow \underline{\underline{\lim_{x\to 0}}} \dfrac{e^x - 1}{x} = 1$.

$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x$$

$$(e^x)' = e^x, \quad (\log x)' = \frac{1}{x}. \quad \boxed{} \longleftrightarrow \text{적분.}$$

$$(x^n)' = n x^{n-1}.$$

$+$ (미분법) $+$ (부분적분).