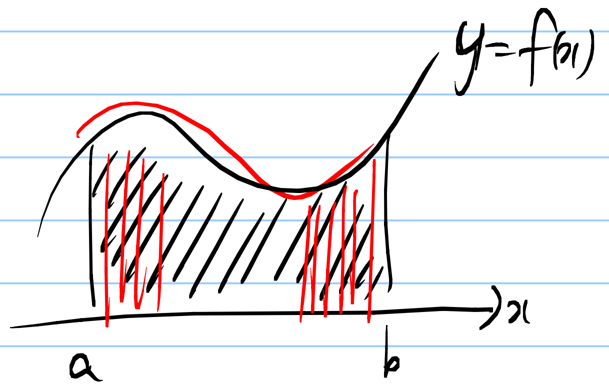
$$f'(x) = \frac{d}{dx} f = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x+h) - f(x)$ → y의 변화량.

h → x의 변화량



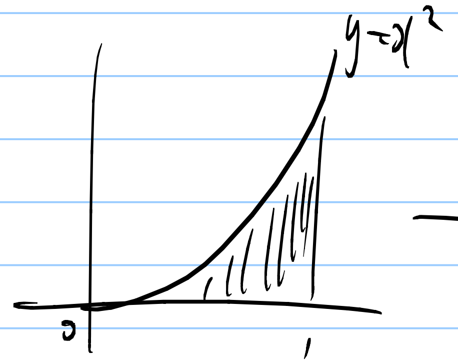
n등분 & $n \to \infty$

$$S_a(b) = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

$$\Delta x = \frac{b-a}{n}, \quad x_k = a + k\Delta x$$

$$= \int_a^b f(x)\,dx$$

$$= F(b) - F(a) \quad \text{where } F'(x) = f(x).$$



$\to \frac{1}{3}$

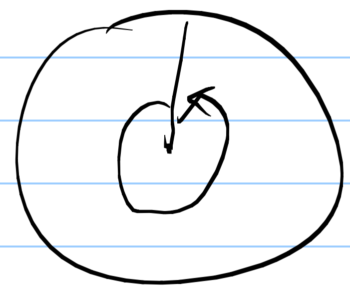
$$\frac{d}{dx} x^n = n x^{n-1} \quad \text{for } n \in \mathbb{R}.$$

$\Rightarrow$ 다항함수.

$$\rightarrow \int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad \text{for } n \in \mathbb{R} - \{-1\}$$

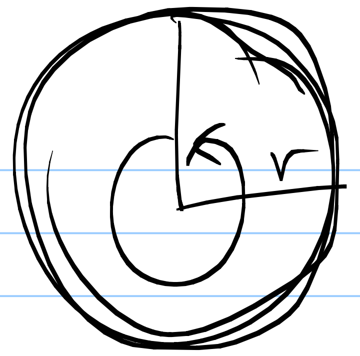
$$\int x^{-1} \, dx = \int \frac{dx}{x} = \, ?$$

15° → degree.



360°

$$\theta = \frac{l}{r}.$$

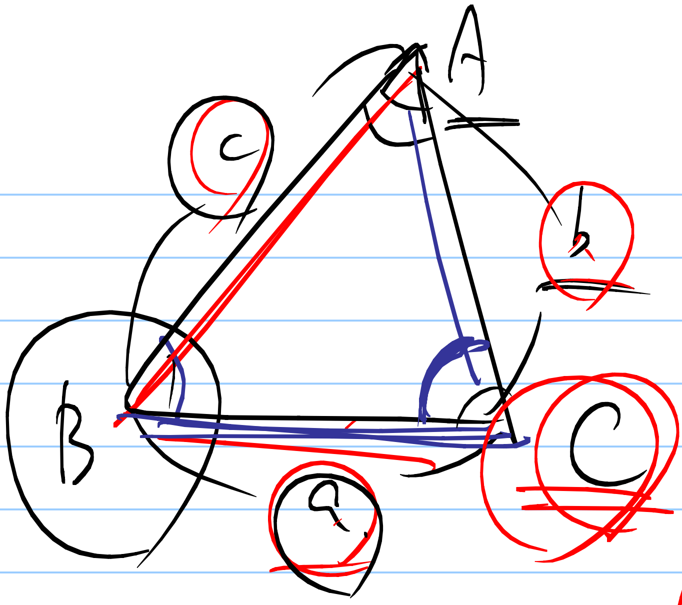


$$\theta = \frac{2\pi r}{r} = \boxed{2\pi = 360^\circ} = 6.28\ldots$$

$$\pi = 180^\circ \Rightarrow \frac{\pi}{180} = 1^\circ$$

$$1 = \frac{180^\circ}{\pi}$$

$$2n\pi + \theta = (2\pi)n + \underline{\underline{\theta}} \quad n \in \mathbb{Z}$$



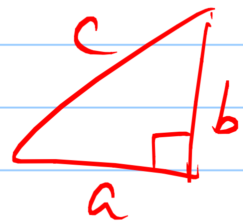
삼각형 결정조건

1) 세변.

2) 두변 & 사잇각.

3) 한변과 세각. $(A+B+C=\pi)$

$$c^2 = a^2+b^2-2ab\cos C.$$



$\Rightarrow c^2=a^2+b^2$

$$a^2 = b^2+c^2-2bc\cos A.$$

$$b^2 = c^2+a^2-2ca\cos B$$

Cos

세변을 알 때 $\Rightarrow$ $a^2 = b^2 + c^2 - 2b\cos A$

$$\Rightarrow \boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

삼각함수의 덧셈정리.

$$\boxed{\sin(x+y)} \neq \sin x + \sin y.$$

$$\sin(60^\circ) = \sin(30^\circ + 30^\circ) \neq \sin 30^\circ + \sin 30^\circ.$$

$$\sin 30^\circ = \frac{1}{2}, \quad \sin 45^\circ = \frac{1}{\sqrt{2}} \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

① $\sin 15^\circ = ?$ $\quad \sin(45^\circ - 30^\circ) = \sin\left(\frac{30^\circ}{2}\right).$

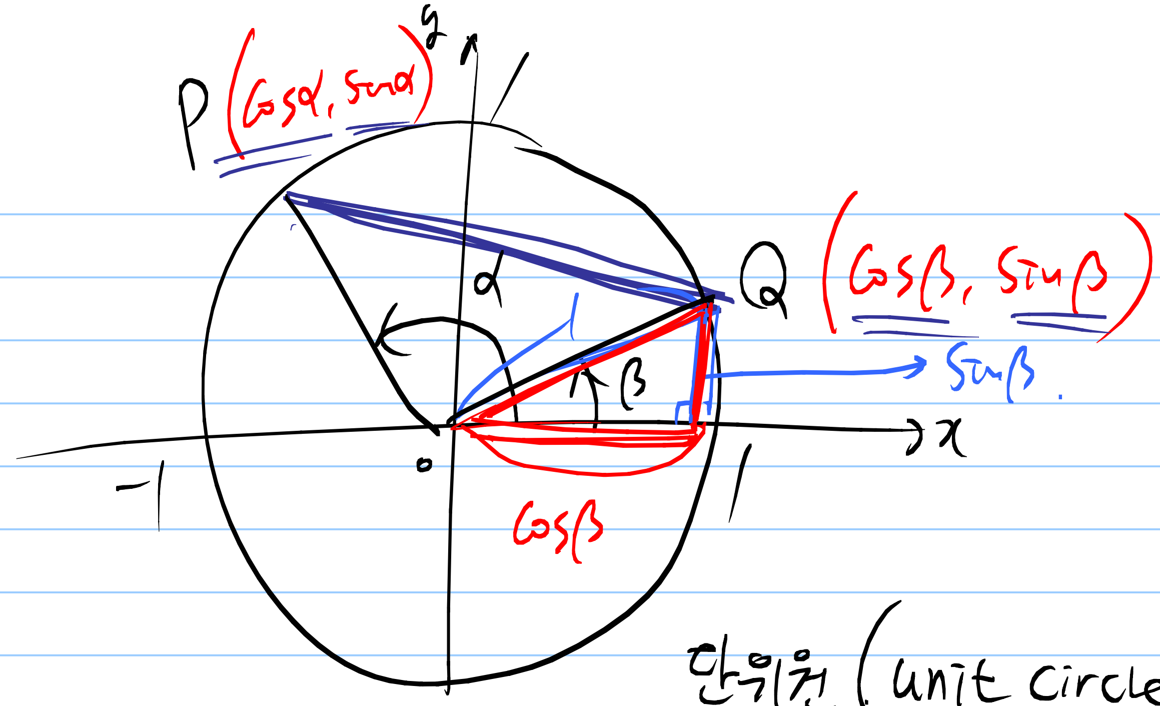
②. 적분 전개 vs. 인수분해.

미분 vs 적분.

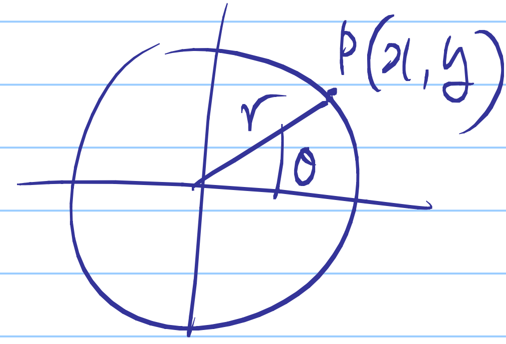
$$\int \sin x \, dx = -\cos x + C.$$

$$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} \, dx.$$

$$\int \sin x \cos x \, dx \Rightarrow \int \left(\sin + \sin \right) dx.$$

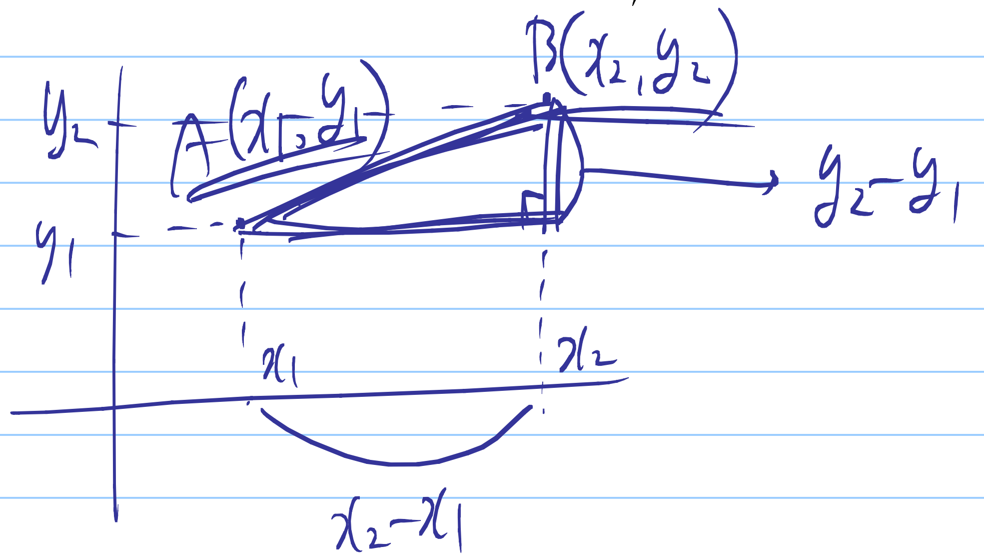


단위원 (unit circle)



$\sin\theta = \frac{y}{r} = y$

$\cos\theta = \frac{x}{r} = x$



$$\overline{AB}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

$$\cos\theta \times \cos\theta = \boxed{\cos^2\theta}$$

$P(\cos\alpha, \sin\alpha)$, $Q(\cos\beta, \sin\beta)$

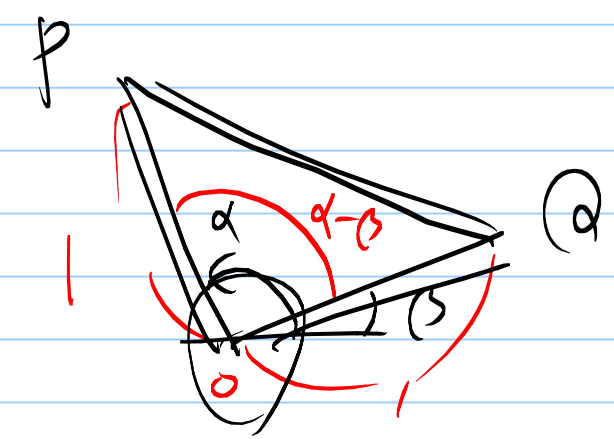
$(\cos\theta)^2$

$\cos(\theta^2)$

$$\overline{PQ}^2 = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$$

$$= \cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta - 2\sin\alpha\sin\beta$$

$$= 2 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta.$$



$$\overline{PQ}^2 = 1^2 + 1^2 - 2\cdot 1\cdot 1\cdot\cos(\alpha-\beta)$$

$$= 2 - 2\cos(\alpha-\beta)$$

$$\therefore \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$$

$\beta \leftarrow -\beta$: $\cos(\alpha+\beta) = \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

$\cos$ = (같은 것의 곱)

$\sin(\alpha+\beta) = ?$ $\quad \cos(\alpha+\beta)$ $\quad \cos\left(\alpha+\beta+\frac{\pi}{2}\right) = -\sin(\alpha+\beta)$

$$\therefore \sin(\alpha+\beta) = -\cos\left(\alpha+\beta+\frac{\pi}{2}\right)$$

$$= -\cos\alpha\cos\left(\beta+\frac{\pi}{2}\right) + \sin\alpha\sin\left(\beta+\frac{\pi}{2}\right)$$

$$= +\cos\alpha\sin\beta + \sin\alpha\cos\beta.$$

$$\boxed{\sin(\alpha+\beta) = \underline{\sin\alpha}\,\underline{\cos\beta} + \underline{\cos\alpha}\,\underline{\sin\beta}}$$

$(\sin) = $ (다른 것의 합)

$\beta \to -\beta$: $\sin(\alpha-\beta) = \sin\alpha\,\underline{\cos(-\beta)} + \cos\alpha\,\underline{\sin(-\beta)}$

$$= \sin\alpha\cos\beta - \cos\alpha\sin\beta.$$

멋있겠니.

$$\underline{\sin(\alpha+\beta)} = \underline{\sin\alpha\cos\beta} + \underline{\cos\alpha\sin\beta}.$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

$\alpha = \beta$:

$$\sin 2\alpha = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha = 2\sin\alpha\cos\alpha .$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha .$$

$$= 1 - \sin^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha$$

$$= \cos^2\alpha - 1 + \cos^2\alpha = 2\cos^2\alpha - 1 .$$

$\Rightarrow \alpha \to \frac{x}{2}$: $\cos x = 1 - 2\sin^2\frac{x}{2} = 2\cos^2\frac{x}{2} - 1 .$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} , \quad \cos^2\frac{x}{2} = \frac{1+\cos x}{2} .$$

$$\sin 15^\circ = \sin\left(\frac{30^\circ}{2}\right) = \sin(45^\circ - 30^\circ)$$

$\sin 75^\circ = \cdots$

$$\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)$$

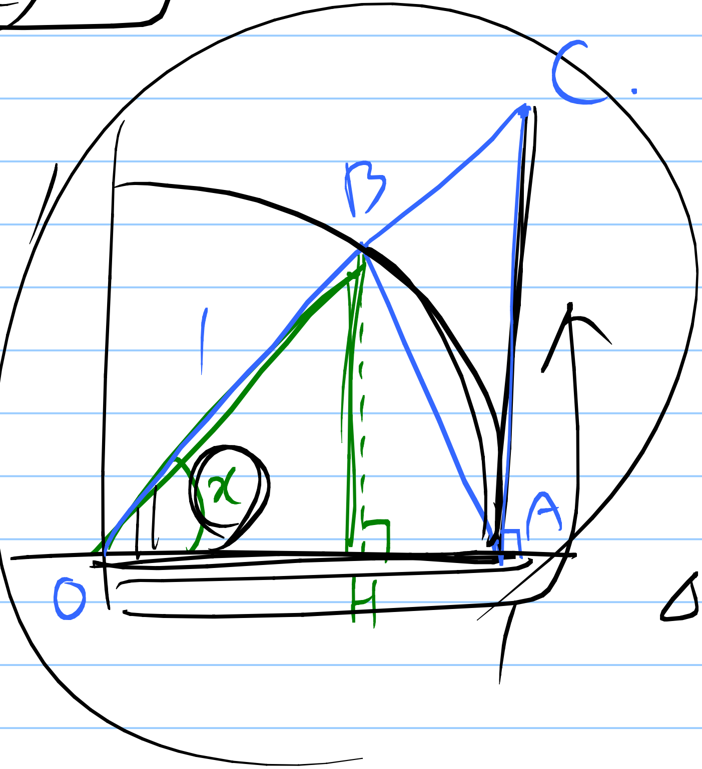
$\lim_{x \to a} f(x)$: $x=a$ 주변부의 값 $(x \neq a)$.

$f(a)$

$$\lim_{x \to 0} \frac{\sin x}{x} \qquad \frac{0}{0}.$$

$\sin x = \overline{BH}.$

$\tan x = \overline{AC}.$



$\triangle OAB = \frac{1}{2} \times 1 \times \sin x.$

$\triangle OAB = \frac{x}{2\pi} \cdot \pi.$

$= \frac{1}{2} x$ x : rad.

$\triangle OAC = \frac{1}{2} \times 1 \times \tan x.$

$= \frac{1}{2} \frac{\sin x}{\cos x}.$

$$\triangle OAB < \triangle OAB < \triangle OAC \Rightarrow \cancel{\frac{1}{2}} \sin x < \cancel{\frac{1}{2}} x < \cancel{\frac{1}{2}} \frac{\sin x}{\cos x}.$$

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x} = 1}$$

$x \to 0$

$$\frac{\sin x}{x} < 1 < \frac{\sin x}{x}\frac{1}{\cos x}.$$

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x} \to 1 \qquad \cos 0 = 1.$$

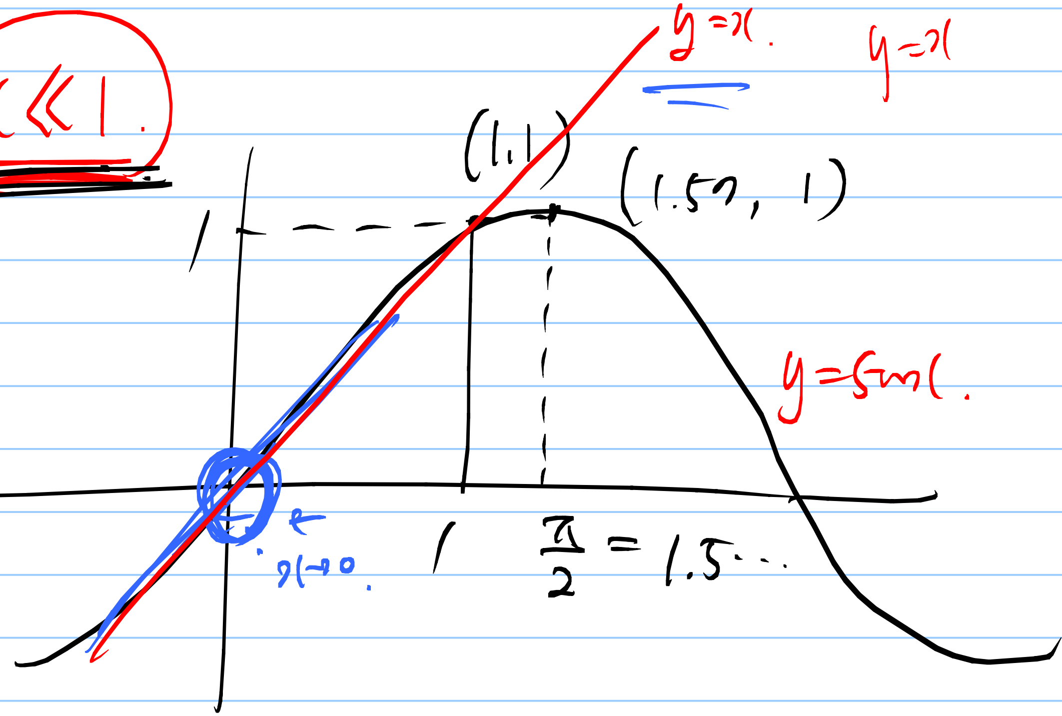
$$\frac{1}{\cos x} \to 1$$

$1.$

$\sin x \approx x$ for $x \ll 1.$

Taylor expansion.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$



$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

$$e = \boxed{\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n}} = 2.718281828\ldots$$

$$\parallel$$

무리수 $= (1+0)^{\infty}$ $\qquad \frac{1}{n} = x.$

exponential.

(natural)

$$\boxed{e = \lim_{x \to 0} (1+x)^{\frac{1}{x}}}$$

$$= 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

$$\boxed{e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots} = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

definition.

$n! = 1 \times 2 \times \cdots \times n.$

(factorial).

$$\lim_{x \to 0} \frac{\underbrace{e^x - 1}_{= y}}{x} = ? \ \frac{0}{0}.$$

$e^x = 1 + y.$

$\boxed{x = \log_e (1+y).}$

$\log_a x$

$$= \lim_{y \to 0} \frac{y}{\log(1+y)} = \lim_{y \to 0} \frac{1}{\frac{1}{y}\log(1+y)}$$

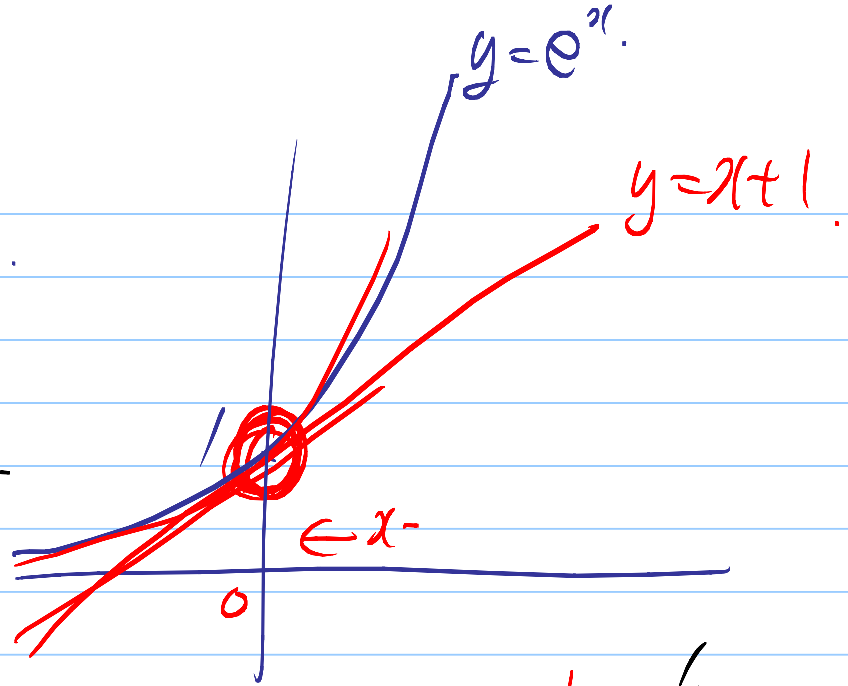
$$= \lim_{y \to 0} \frac{1}{\log (1+y)^{\frac{1}{y}}} = \frac{1}{\log_e e} = 1.$$

$$\boxed{\lim_{x \to 0} \frac{e^x - 1}{x} = 1}$$

cf) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$e^x - 1 \approx x$ for $x \ll 1$.

$e^x \approx 1+x$ for $x \ll 1$.



$\boxed{e = 2.71828\cdots}$

$$e = \lim_{x \to 0} (1+x)^{\frac{1}{x}}$$

$$\lim_{x \to 0} (1+2x)^{\frac{1}{x}} = \left[\lim_{2x \to 0} (1+2x)^{\frac{1}{2x}}\right]^{\times 2} = e^2.$$

- 덧셈정리

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

- 극한

$$\lim_{x\to 0}\frac{\sin x}{x} = 1, \qquad e = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$$

$$\Rightarrow \lim_{x\to 0}\frac{e^x-1}{x} = 1.$$

$$f'(x) = \frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$(\sin x)' = \frac{d}{dx} \sin x = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \overset{\to 1}{\cos h} + \cos x \sin h - \sin x}{h}.$$

$$= \lim_{h \to 0} \left[\frac{\sin x}{h} (\cos h - 1) + \cos x \cdot \frac{\sin h}{h} \right]$$

$$\cos 2x = 1 - 2\sin^2 x.$$

$$\left(\frac{1 - \cos 2x}{2} = \sin^2 x. \right) \Rightarrow \boxed{\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}.}$$

$$\lim_{x\to 0}\frac{\sin x}{x}=1.$$

$$=\lim_{h\to 0}\left[\frac{\sin x}{h}(-2)\sin^2\frac{h}{2}+\cos x\cdot\frac{\sin h}{h}\right]$$

$$=\lim_{h\to 0}\left[-2\sin x\,\frac{\sin\frac{h}{2}\cdot\sin\frac{h}{2}}{\frac{h}{2}\cdot 2}+\boxed{\cos x}\,\frac{\sin h}{h}\right]$$

$\to 0.$ $\quad 0.$ $\quad 1.$

$$=\cos x.$$

$$\therefore\ \boxed{(\sin x)'=\cos x.}$$

$$(\cos x)'=\lim_{h\to 0}\frac{\cos(x+h)-\cos x.}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \cdot \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \left[\cos x \cdot \underbrace{\frac{\cos h - 1}{h}}_{\to 0} - \boxed{\sin x} \cdot \underbrace{\frac{\sin h}{h}}_{1} \right].$$

$$= -\sin x.$$

$$\therefore \boxed{\begin{aligned} (\cos x)' &= -\sin x. \\ (\sin x)' &= \cos x. \end{aligned}}$$

$$\Rightarrow \int \sin x \, dx = \underline{\underline{-\cos x}} + C.$$

$$\int \cos x \, dx = \sin x + C.$$